TRANSIT-BASED EVACUATION PLANNING UNDER DEMAND UNCERTAINTY

By

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To my parents
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Evacuation planning has drawn a significant amount of attention over the past several decades. Previous studies have focused on different aspects of evacuation planning, however, most deals with auto-based evacuation. Unfortunately, many people in risk areas do not own or have access to private vehicles during evacuation. Recent experiences with Hurricane Katrina and Rita in 2005 have highlighted a need for a plan to evacuate those who depend on public transportation. These include individuals without access to private vehicles, the poor and elderly, tourists and people with medical needs. Despite a great deal of attention and focus towards transit evacuation planning since 2005, deficiencies still remain in the emergency response plans using public transportation. There are very few attempts in the literature on modeling of transit-based evacuation.

On the other hand, evacuation planning is challenging due to the uncertainties involved. There can be numerous sources of uncertainties in planning an evacuation and most of them are not easy to quantify. The majority of existing evacuation planning models does not account for the uncertainty during evacuation. Ignoring uncertainties
during evacuation can result in some serious undesirable difficulties during the entire process and the whole evacuation could end up with a failure. There is a pressing need for models that incorporate uncertainty and thus assure viable measures for the unanticipated.

The main purpose of this dissertation is to develop a plan that evacuates those who depend on public transportation to safe areas reliably and as fast as possible during an emergency. In this dissertation, optimization models are developed for transit-based evacuation planning which are more efficient and reliable in the face of demand uncertainty. Demand uncertainty refers to the uncertainty associated with the number of transit-dependent evacuees. Two critical decision-making components of transit evacuation planning are studied, namely, public shelter locations and pick-up locations (staging areas). Considering that it is practically difficult to obtain the probability distribution of the uncertain demand, a set based robust optimization approach is adopted to address the demand uncertainty in the proposed models.

Three models considering different aspects of transit evacuation are developed in this dissertation. The first model determines the optimal locations of public shelters and their capacities in the region, from a given set of potential sites for a large-scale transit evacuation (i.e., regional level planning). The other two models focus more on transit evacuation planning on a local level, i.e., from the perspective of a transit agency in a particular county. In the second model, a robust optimization model is proposed to locate the pick-up points for evacuees to assemble, and allocate available transit vehicles to transport the assembled evacuees between the pick-up locations and different public shelters. For this model, the locations of public shelters are assumed to
be given and known. The last model considers an integrated approach to model pick-up and shelter locations simultaneously. In particular, it simultaneously determines the optimal shelter locations, pick-up locations, allocation of given transit vehicles and capacities of public shelters. Efficient solution algorithms based on cutting plane schemes are proposed to solve the models.

The computational results illustrate the effectiveness of the proposed models. Robust plans achieve nearly the same level of reliability (in terms of meeting the evacuee demand) with a significant reduction in evacuation time as compared to a conservative plan, i.e., one based on the worst-case demand. The importance of considering demand uncertainty in evacuation planning is also demonstrated. The resulting plans obtained from the proposed model are robust with respect to different demand scenarios and effective in real-world mass evacuations. Overall, the proposed models are very useful for the planners and emergency managers for a more reliable and efficient transit-based evacuation planning.
CHAPTER 1
INTRODUCTION

1.1  Background

Natural and man-made disasters occur frequently and have always been a major concern to humanity and a threat to lives and property. The most commonly encountered natural disasters include hurricanes, earthquakes, and floods, while nuclear meltdowns, hazardous-material spills, and terrorist attacks comprise the primary man-made threats. Despite their differences, similar issues arise in the aftermath of these events. It is essential that we use the knowledge gained from the past disasters to develop effective plans and avoid loss of life and damage to infrastructure in the future ones. Aiming to reduce adverse consequences of these disasters, the field of Emergency Management has evolved substantially over the years (McLoughlin, 1985).

Evacuation is often the most viable response action for protecting affected people. Evacuation is defined as “mass physical movements of people, of a temporary nature, that collectively emerge in coping with community threats, damages, or disruptions” (Quarantelli, 1998). Recent trends illustrate an increase in vulnerability towards these disasters. According to Federal Emergency Management Agency (FEMA), there are 45 to 75 major emergency incidents annually in the United States that require evacuation (FEMA, 2008). Another recent study of evacuations by the Sandia National Laboratories (Jones et al., 2008) showed that between 1990 and 2003, 230 evacuations involved 1,000 or more evacuees. Figure 1-1 shows the evacuation frequency based on the hazard type and Figure 1-2 shows the evacuation frequency based on the evacuating population size. Some of these evacuations are large-scale events (e.g., evacuations in response to hurricanes), potentially affecting millions of people.
Figure 1-1. Evacuation frequency based on hazard type (1990-2003) (Source: Jones et al., 2008)

Figure 1-2. Evacuation frequency based on evacuating population size (1990-2003) (Source: Jones et al., 2008)

Over the past several decades, interest in and awareness of the topic of evacuation has grown enormously. There are various studies that focus on different
aspects of evacuation planning such as evacuee behaviors (e.g., Johnson and Zeigler, 1986; Baker, 1991; Whitehead et al., 2000; Fu and Wilmot, 2004), traffic control strategies (e.g., Wolshon, 2001; Theodoulou and Wolshon, 2004; Chen et al., 2007), sheltering site selection (e.g., Sherali et al., 1991; Kongsomsaksakul et al., 2005), demand management (e.g., Sbyati and Mahmassani, 2006; Chen and Zhan, 2008), and real-time route guidance (e.g., Chiu et al., 2006; Yuan et al., 2006; Liu et al., 2007) etc. However most of these studies are for auto-based evacuation. Unfortunately, not all the people in risk areas will own or have access to personal vehicles during evacuation.

Recent experience with hurricane Katrina and Rita in 2005 have highlighted the need of evacuation planning for transit-dependent peoples (e.g., Litman, 2006; Renne et al., 2008; TRB, 2008). More specifically, during hurricane Katrina, there was no effective plan to evacuate those who were dependent on the public transportation. The plans lacked important details such as how to use the buses, availability of drivers, bus routes or the pick-up locations (staging areas) to collect evacuees (USDOT & USDHS, 2006). Evacuation planning during Katrina was relatively effective for people with automobiles but failed for transit-dependent residents (Litman, 2006). State and local plans during hurricane Rita appeared to be better coordinated in providing public transportation during the evacuation. However, the plan still lacked many details concerning the evacuation of transit-dependent residents with special needs (USDOT & USDHS, 2006).

It is believed that the public transit could have played an important role in assisting the evacuation of carless residents in such emergency situations, if it had been properly integrated in the evacuation plans (TRB, 2008). The potential role of transit is even
more evident in states and cities that have high percentage of carless residents, as is the case with various cities in the United States (Renne et al, 2008). The extra risks faced by carless population during an evacuation are well recognized and thus transit evacuation planning has been recently viewed as an important component of evacuation planning and management.

Interest in the topic of transit evacuation has increased significantly since 2005. Despite a great deal of attention and focus towards transit evacuation planning, deficiencies still remain in the state and local emergency response plans to evacuate using public transportation (Bailey et al., 2007; Hess and Gotham, 2007; TRB, 2008). Evacuation planning continues to focus more heavily on auto-based strategies while only a little emphasis on transit-based evacuations. In fact, there are a limited number of studies on modeling of transit-based evacuation. This dissertation aims to address this critical gap.

On the other hand, evacuation planning is very challenging due to the uncertainties involved during evacuation. There can be numerous sources of uncertainties and most of them are not easy to quantify. However, the majority of existing evacuation planning models does not account for the uncertainty during evacuation. Deterministic models cannot reliably represent a real-life evacuation under uncertain conditions. Therefore, for more effective evacuation planning, it is imperative that plans account for uncertainties. There is a pressing need for models that incorporate uncertainty and thus assure viable measures for the unanticipated.

This dissertation approaches the transit-based evacuation problem from a robust planning perspective and aims to facilitate the transit-based evacuation in response to
various disasters and catastrophic events. The objective of this dissertation is to develop a plan that evacuates those who depend on public transportation to safe areas reliably and as fast as possible during an emergency. In this dissertation, optimization models are developed for transit-based evacuation planning that are more efficient and reliable in the face of demand uncertainty. Two critical decision-making components of transit evacuation planning are studied, namely, public shelter locations and pick-up locations (staging areas) and demand uncertainty in the models refers to the uncertainty associated with number of transit-dependent evacuees.

1.2 Dissertation Outline

This dissertation is organized as follows. Chapter 2 reviews the existing literature on evacuation planning, the role of transportation in evacuation planning, transit evacuation planning and evacuation planning under uncertainty. The next three chapters present different models for transit-based evacuation planning. Chapter 3 develops a robust model for optimal shelter locations and their capacities for regional transit evacuation planning with demand uncertainty. Chapter 4 proposes a model for optimal transit pick-up locations and allocation of transit vehicles in a county or region. Chapter 5 then presents an integrated approach to determine optimal shelter and pick-up location for transit evacuation planning under demand uncertainty. Finally conclusions and suggestions of future research work are provided in Chapter 6.
CHAPTER 2
LITERATURE REVIEW

Evacuation planning has drawn a significant amount of attention over the past several decades. There is an extensive literature on evacuation planning and management spanning various disciplines. In this chapter, we will start with the current understanding of the overall emergency planning and management process and practice. Next, we will summarize the role played by transportation systems in supporting and assisting evacuation during an emergency response. Different roles, practices and strategies will be discussed in brief. We then turn our focus to transit evacuation planning, which aims at assisting transit-dependent populations during evacuation. Given that there are substantial uncertainties involved in evacuation planning, we lastly review the studies on evacuation planning under uncertain conditions.

2.1 Emergency Planning and Management

In the event of a natural or man-made disaster, emergency preparedness plays a vital role in ensuring the safety of residents and transportation system. Emergency preparedness is defined as “the preparation of a detailed plan that can be implemented in response to a variety of possible emergencies” (Sisiopiku et al., 2004). Emergency preparedness for a state or locality is often measured in terms of its ability to respond to an emergency in a timely and effective manner. Transportation officials have a very important role to play in preparing for emergencies and need to work closely with emergency management agencies and other stakeholders to promote a smooth functioning of transportation system during or following an emergency. According to the U.S. Department of Transportation, “if a community is not actively planning to optimize
the operation and coordination of its transportation system during natural disasters or national security events, there’s a missing link in their emergency preparedness plans" (USDOT, 2003).

In preparing transportation systems for emergencies, it is helpful to understand the emergency management planning process. The major components of an emergency response and management plan are mitigation, preparedness, response and recovery (Nakanishi et al., 2003; Sisiopiku et al., 2004). The mitigation step of the emergency planning process refers to the development of measures to reduce or minimize the risk of damage in the event of an emergency situation. Preparedness refers to the development of emergency response plans and decision making guidelines in advance and it should focus on designating roles and responsibilities to the agencies, implementing communication systems and protocols; developing emergency training and drills to maintain readiness; and revising and updating plans. The response phase can be defined as taking actions (e.g., allocating resources, implementing evacuation orders etc.) when an emergency situation takes place in order to save lives and reduce damage. The recovery phase consists of efforts to return evacuees to affected areas and restore the economic, physiological, and social life of a population back to its normal condition.

2.1.1 Evacuation Management Process

This section introduces the general framework for the evacuation management process after the detection of any event that poses a disaster threat for a region. The first step after the detection of disaster is hazard analysis and forecasting to understand the type of disaster and its impact. Next, vulnerability analysis is conducted to assess how many people are at risk and what is the expected damage to the infrastructure. The
first decision is whether the situation needs disaster response or not. If the disaster is not going to affect people in the region and they can remain at their homes without harm, then no evacuation action may be needed other than sending people home.

If the event is characterized as a major disaster, then the disaster response needs to be invoked. Evacuation is often the most viable response action for handling emergency situation during many disasters. Appropriate actions for evacuation are determined at this stage. The main tasks required for evacuation operations are shelter analysis, traffic management, evacuation of people with special needs, continuous monitoring of the event and network conditions, and establishment of effective public communication for the sharing of necessary information.

During the disaster response, evacuation management operations include a variety of tasks requiring different departments and agencies e.g., firefighting, medical services, department of transportation, police etc., to work together to facilitate the evacuation. During evacuation, specific government agencies are assigned different emergency support roles. Once the state of emergency is over, response actions are replaced mostly by recovery actions.

2.1.2 Emergency Response Plan

The federal government, through FEMA, requires every state to have a comprehensive emergency response plan. These plans guide emergency operations for all types of hazards and their specifics depend on the regional characteristics of the geography and transportation system. Most states take a two-tiered approach to emergency planning and response. Primarily, evacuation planning, response, and recovery activities are developed at the local (e.g. county or city) level. State level
emergency management (EM) agencies typically serve to coordinate local transportation agencies, EM agencies, and law enforcement.

Federal, state and local officials have recognized the need for better evacuation planning after some recent disasters (e.g., Hurricane Katrina and Rita) in 2005 and they have taken initiatives and made changes to address the lessons learned. Some of the key elements of evacuation planning and implementation which should be addressed in a state emergency response plan includes, decision making and management; planning; public communication and preparedness; evacuation of people with special needs; evacuation operations; sheltering; training and exercises. For more detailed information about these key elements, please refer to USDOT & USDHS (2006).

2.1.3 Evacuation Planning

Evacuation planning is a very challenging part of an emergency management plan. The visible part of the evacuation process occurs when people get to the road trying to reach safe destinations. However, in order to accomplish a successful evacuation, much planning is necessary before this last step is taken. The sequence of activities that leads up to an evacuation order is typically led and coordinated by state and local level emergency management officials.

State and local government agencies must properly coordinate the movement of evacuating vehicles. There must be better integration between evacuation plans at the state and local levels and greater real-time coordination between state and local governments while managing an evacuation. For instance, in order to avoid future massive traffic jams and coordination problems, state and local governments must be on the “same page” in executing contra-flow operations. Their plans must be seamlessly integrated so that when the need for a mass evacuation arises, these government
entities can work together based on the same pre-existing, shared plan. Furthermore, as a mass evacuation usually affects a population dispersed over a wide geographical area, a number of governmental jurisdictions must work together in order to improve the mobility of different transportation modes across their boundaries during an evacuation.

An evacuation plan is the outcome of a planning process and cannot be created by one agency. Plans for a complex, multi-jurisdictional disaster require coordination and integration of plans between federal, state, and local agencies as well as NGOs and the private sector. Evacuation plans that are integrally linked with the plans of supporting agencies must be updated regularly to reflect changes in capabilities and resources of the partners.

The evacuation plans evaluate the risk to the community; provide the information that emergency managers need to execute an evacuation; determines what actions will be taken and who will carry out those actions; and discuss strategies and alternatives if events do not go as anticipated.

2.1.4 Emergency Management Roles in Evacuation

Historically, evacuation planning has been the responsibility of emergency management and law enforcement agencies. In response to the problems experienced during some of the recent disaster events, many state departments of transportation have begun to take a more active role in evacuation planning. In current practice, evacuation planning involves not only emergency managers and law enforcement officials, but also transportation officials and officials from other agencies, because of their expertise in different aspects of evacuation planning. Under the emergency response plan, specific governments and agencies are assigned different emergency
support roles. For the details on roles during an emergency and their primary responsible agencies, refer USDOT & USDHS (2006).

Evacuations are typically ordered and managed by local officials. In case of a mass evacuation, where multiple counties (or states) are concerned, state-level agencies become involved in the evacuation process. Although federal agencies like FEMA are involved in some aspects of evacuations, the role of federal agencies during an evacuation is typically limited to providing assistance to state and local agencies as the conditions of the emergency surpass local resources and capabilities. Non-government organizations, such as American Red Cross also play an important role in disaster response and provide shelter, food, and health services to address basic needs.

2.1.5 Authority and Command Structure

Command structure is one of the key elements that must be specified in emergency response plans. Command is defined as the authority to issue evacuation orders and every state has a different command structure. By law, governors in most states have the ultimate authority to order an evacuation in case of a disaster. However, this authority is often delegated to local officials (e.g. mayor, city council, county law enforcement, county judge and county president). At the same time, local officials rely on county or municipal emergency managers for advice. This is primarily because emergency managers have better knowledge and training in the evacuation process and are better informed on current local conditions. Details about major components of state and federal level command structure can be found in USDOT & USDHS (2006).
2.2 Role of Transportation in Evacuation Planning

Although history has shown that evacuation planning has been addressed relatively less within the transportation community throughout the years, interest and involvement of transportation professionals in the field have grown considerably in the past decade. In response to the difficulties experienced during some of the past evacuations, many Departments of Transportation (DOT's) have begun to take a more active role in planning, management and operations of evacuations.

Evacuations are dependent on the ability of transportation systems to move evacuees out of the affected region. The transportation system plays an important role in supporting and assisting evacuations during an emergency response. Transportation personnel are involved before, during and after evacuations by managing and maintaining transportation systems, including traffic control, monitoring, planning, and management.

Among the best defined and well developed roles of transportation professionals in evacuations are in the area of direction and control of highway networks. Transportation network congestion is one major problem encountered during an evacuation, and it can be catastrophic and life threatening. Therefore, better utilization of available resources and the development of effective strategies are essential to manage traffic and lead people to safety during evacuation. Numerous traffic control strategies have been developed to facilitate evacuations, e.g., contraflow operations (e.g., FEMA, 2000a; PBS&J, 2000; Wolshon, 2001), phased evacuations (e.g., Chiu, 2004; Tuydes and Ziliaskopoulos, 2005; Mitchell and Radwan, 2006; Sbayti and Mahmassani, 2006) and arterial signal control (e.g., Sisiopiku et al., 2004; Chen, 2005; Chen et al., 2007). These strategies help optimize the throughput of evacuees during evacuation.
Transportation professionals also focus on support activities such as providing information on open and closed roadways and detours; committing manpower and material resources for roadway closures; performing inspections, repairs, and debris removal on affected roadways; and using Intelligent Transportation Systems (ITS) to provide en route guidance and communications and traffic flow monitoring.

Transportation planners also play a key role in the process of evacuation demand modeling, which is an important and valuable aspect of evacuation planning in order to quantify the travel patterns of evacuees and to assess the transportation related problem that may hinder an evacuation. Behavioral models for evacuation demand have received a significant amount of attention for many years. Behavioral aspects of evacuees during evacuation include how and when they chose to evacuate and choice of their routes and destinations. The difficulty with these demand models lies in the complexity of human psychology. Researchers have examined a wide range of factors that affect evacuation behavior and demand (Baker, 1991; Dow and Cutter, 1997; Dash and Gladwin, 2007 for a detailed review). Current practice in evacuation planning to estimate time dependent travel demand is a two-step process. In the first step, the total number of people expected to evacuate is estimated using participation rates. These rates are determined by the different characteristics of the disaster and the factors affecting evacuee behaviors. In the second step, the time at which the evacuees are expected to evacuate is estimated. This is typically done using a response (loading) curve which estimates the percent of total evacuation demand which will evacuate in each time period. For a more detailed review on studies related to evacuation demand modeling, refer Yazici and Ozbay (2008) and Pel et al. (2011).
The review of transportation's role in emergency evacuation demonstrates the importance of and contributions made by transportation agencies in supporting and assisting evacuations. Over the past several decades, the role of transportation in evacuation has significantly expanded. In the United States the transportation system is dominated by the automobile. As a result, most of the evacuation planning is based on auto-based strategies. Unfortunately, not all the people in risk areas will own or have access to personal vehicles during evacuation. Evacuation planning for these carless and low-mobility populations, who are dependent on public transportation to evacuate during disaster, has been largely ignored. Recently, there is increasing interest in the role of public transportation during evacuation and some efforts have been made towards the use of transit in assisting evacuation. In the following section, transit-based evacuation planning for transit-dependent residents will be reviewed.

2.3 Transit Evacuation Planning

Public transit agencies are capable of providing assistance during crisis situations, performing services such as evacuating victims and transport of emergency personnel etc. In the aftermath of major disasters, public transit has often maintained the mobility of residents faced with damaged or blocked roadway conditions. Successful application of public transit in some of the incidents from recent years highlights the critical role of transit systems:

- During the San Francisco earthquake of 1989, some primary arterials of the area were destroyed, which closed connection to the other roadways within San Francisco and connections with Oakland, making travel in the city difficult. Within a few hours of the earthquake, the BART subway system was running and providing reliable transport in the city. Later, other transit agencies in the area also joined in to help restore the transportation system (Public Transit, 1992).

- Harrisburg, Pennsylvania’s Capitol Area Transit (CAT) responded to a variety of emergency conditions during the blizzard of 1996 and its aftermath. CAT vehicles...
and employees made significant contributions to Harrisburg’s winter storm response and recovery by, for example, evacuating residents from flood zones and transporting firefighters during a four-alarm fire (Harrisburg, 1996).

- Transportation was provided by Orlando’s LYNX transit agency for the victims of tornado in February 1998. In April, a tornado in Nashville led to similar response by Nashville’s Metropolitan Transit Authority (Berlin, 1998).

- Public Transportation played a critical role in the evacuation of people from Manhattan, NY during the terrorist attack of September 11, 2001. Within minutes of the attacks, New York City Transit went to emergency operations and dispatched hundreds of buses to successfully evacuate people from the World Trade Center area (Jenkins and Winslow, 2003).

2.3.1 Transit System Roles in Emergency Evacuation

Historically, trains and buses have played an important role in the evacuation. Zelinsky and Kosinski (1991) examined twenty-seven major evacuations that have occurred in the past fifty years. According to this study, trains and buses were important modes in 20 of the 27 studied evacuations. In ten of these evacuations, the majority of people used trains and buses.

Public transit is an extremely versatile and flexible mode of transportation that can play a vital role in supporting evacuations during emergency situations. Transit can play multiple roles in an emergency evacuation, and these roles depend on the nature of the disaster and its location, availability of vehicles and operators at the time of disaster, and the extent of damage, if any, to transit equipment and facilities.

The case studies conducted in TRB (2008), illustrate the role that transit can play in an emergency evacuation. Transit can play a critical role in evacuating people without access to private vehicles, moving them out of the affected area to safe shelter locations. Transit can also be used to transport emergency personnel and equipment’s to an incident site. Finally, transit can also be involved in the recovery phase, bringing the evacuees back to the area and resuming normal transport services for the residents.
2.3.2 Lack of Transit Planning in Emergency Response

Use of public transit in emergency situations is not well studied in the literature and the failure of evacuating residents without access to private transportation, has highlighted the need for urban evacuation and planning for transit-dependent people. Transit-dependent citizens include people without access to private vehicles, the poor and elderly, tourists and people with medical needs.

The evacuation of New Orleans during hurricane Katrina was one of the most significant evacuations in United States history and it was both a success and a miserable failure. Years of planning and coordination amongst transportation planners, emergency managers, and police led to an effective contraflow system that enabled everyone with a car to evacuate. Unfortunately, much of this hard work went unnoticed because national attention focused on the glaring failures in the evacuation effort, particularly the failure to evacuate a significant number of carless people who were unable to leave the city. State and local plans did not properly address the evacuation of those who were dependent on public transportation modes. Although various emergency preparedness groups including the Red Cross, the New Orleans Public Health Department, and the city’s Office of Emergency Preparedness, were arranging the use of Amtrak and city buses for the evacuation of public-transit-dependent people even before Katrina struck, yet these plans lacked important details such as how to use the buses, availability of drivers, bus routes or the pick-up location (staging areas) to collect evacuees (USDOT & USDHS, 2006).

State and local plans during hurricane Rita appeared to be better coordinated in providing public transit during the evacuation of residents. Houston METRO, which assisted greatly during the evacuation, possessed a robust fleet of buses, rail and lift
vehicles. However, the plan still lacked many details concerning the evacuation of special needs residents (USDOT & USDHS, 2006).

It is evident that there are serious deficiencies in government plans to evacuate using public transportation. The Victoria Transport Policy Institute describes New Orleans’s public transport evacuation as follows: “…bus deployment was ad hoc, implemented by officials during the emergency without a detailed action plan… Katrina’s evacuation was relatively effective for people with automobiles but failed transit-dependent residents” (Litman 2006).

TRB (2008) evaluated the potential role of transit systems in accommodating the evacuation in times of emergency. As a part of the study, a literature review was conducted to address the question of the extent to which transit is incorporated into local emergency evacuation plans. According to the findings of this report, the majority of emergency operation plans are only partially sufficient to conduct a successful evacuation and few focus on the role of transit. Transit has a unique role to play in evacuating the carless and people with special needs during an emergency. However, these groups are inadequately addressed in most local emergency evacuation plans (TRB, 2008). In a national survey of hurricane evacuation plans (Wolshon et al., 2001); it was found that state departments of transportation (DOTs) ignored the special needs population. This report also found that most of the cities do not have a sufficient number of buses to evacuate all special needs evacuees.

Hess and Gotham (2007) studied counties in rural New York and found that multimodal evacuation planning was not seriously considered in most evacuation plans and that there are inadequate public transportation resources for mass evacuation.
Bailey et al. (2007) surveyed the emergency response and evacuation plans in 20 metropolitan areas with higher than average proportions of minorities, low income levels, limited English proficiency, and households without vehicle access. It was found that only a few agencies had included transit-dependent population in their emergency plans.

2.3.3 Need for Transit Evacuation Planning

In the United States, New Orleans is not unique in terms of its high percentage of carless residents. In fact, according to U.S. Census (2000), seven cities, including New York (56 percent), Washington D.C. (37 percent), Baltimore (36 percent), Philadelphia (36 percent), Boston (35 percent), Chicago (29 percent) and San Francisco (29 percent), had carless populations higher than the 27 percent found in New Orleans. In addition, the people who own an automobile may also need to rely on other modes due to some other constraints for e.g., the elderly people who have cars may be reluctant to drive them during an evacuation. These demographics face higher risk during disasters. For instance, 71% of those who died during Katrina in New Orleans were over the age of 60 and 47% over the age of 75 (AARP 2006a and 2006b).

The extra risks faced by carless households during an evacuation are well-recognized and have been documented in numerous reports (e.g., Fischett, 2001; Bourne, 2004). Despite the attention, relatively little has been done to improve the situation and some serious deficiencies in the evacuation planning involving public transit remain throughout the United States. Based on the current level of public transit emergency planning and preparedness, it is quite likely that unfortunate tragedies seen in New Orleans during and after hurricane Katrina are bound to repeat unless better practices can be developed and widely adopted (Jenkins et al., 2007).
For detailed information about the role of public transportation during evacuation and evacuation planning for transit-dependent residents, please refer to Higgins et al., 1999; Renne et al., 2008; and TRB, 2008.

2.3.4 Optimizing the Use of Transit in Evacuation

Before 2005, public transportation operators in the United States did not play a strong role in evacuation planning, nor were they viewed as a viable option for evacuation. Now, there is increased national awareness and interest in the role of public transportation in evacuation.

Ever since hurricanes Katrina and Rita in 2005, the Federal Government has increased the efforts to assist state and local governments in improving the effectiveness of buses and trains to evacuate special needs populations from disaster areas. The Federal Transit Administration (FTA) has been working with the American Bus Association and the American Public Transportation Association (APTA) to identify and meet the resource requirements of transit agencies in evacuating special needs populations. APTA also created an Emergency Preparedness Task Force to develop strategies for improving coordination between all levels of government in emergency response.

Recent research shows some contributions by the researchers in the area of transit evacuation planning using simulation model. However, optimizing the use of public transit during evacuation events has not received as much attention in the literature.

Interest began recently with Margulis et al. (2006), which developed a deterministic decision-support model to maximize the number of people evacuated during a hurricane by determining the optimal bus allocation for specific pick-up-point-
to-shelter routes. The model execution was limited to a small scale evacuation because of its inability to deal with a large area. The assumption underlying the model is that the pick-up points, number of buses and shelter locations are known and from this the model outputs the number of bus trips for specific pick-up point to shelter routes.

Sayyady (2007) presented an optimization technique to develop an evacuation plan for transit-dependent people during no-notice disasters. A mixed integer linear programming model is developed to generate a transit evacuation routing plan in order to minimize the evacuation time. The assumption underlying the model is that the location of pick-up points and shelters are known and transit vehicles will perform only one trip considering the limitation on evacuation time during no-notice disaster. Model outputs are the expected number of vehicles required, the expected number of citizens evacuated within a given time period, and the evacuation routes for transit vehicles.

Song et al. (2009) presented an optimization modeling technique to develop an evacuation plan for transit dependent residents during hurricane evacuation. The proposed model is formulated as a location-routing problem with uncertain demand. Pick-up points, shelter locations and number of buses are known and the goal is to optimize the bus routing during evacuation in order to minimize the total evacuation time. Abdelgawad et al. (2009) presents an approach to optimally operate the available capacity of mass transit to evacuate transit-dependent people during the no-notice evacuation of urban areas. They proposed an extended vehicle routing problem incorporating constraints for the number of depots, evacuation time frame, and pick-up and delivery locations of evacuees. A framework, using constraint programming was developed to model and solve the problem. A hypothetical evacuation event was
considered and the results demonstrate the optimal scheduling and routing for the buses. Figure 2-1 provides a summary of the reviewed modeling studies for transit evacuation planning.

<table>
<thead>
<tr>
<th>Study</th>
<th>Modeling Approach</th>
<th>Model Output</th>
<th>Objective</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Margulis et al., 2003)</td>
<td>transit allocation plan</td>
<td>number of bus trips</td>
<td>maximize the number of people evacuated</td>
<td>number of buses, pick-up points and shelter locations are known, buses will make trip between specific pick-up and shelter location</td>
</tr>
<tr>
<td>(Sayyady, 2007)</td>
<td>transit evacuation routing plan</td>
<td>expected number of vehicles required and evacuation routes</td>
<td>minimize the evacuation time</td>
<td>pick-up points and shelter locations are known, transit vehicles will perform only one trip</td>
</tr>
<tr>
<td>(Song et al., 2009)</td>
<td>transit evacuation routing plan</td>
<td>evacuation routes for transit vehicles</td>
<td>minimize the evacuation time</td>
<td>number of buses, pick-up points and shelter locations are known</td>
</tr>
<tr>
<td>(Abdelgawad et al., 2009)</td>
<td>transit evacuation scheduling and routing plan with multi-depots</td>
<td>scheduling and routing plan for transit vehicles</td>
<td>minimize the evacuation time</td>
<td>number of buses, pick-up points and shelter locations are known</td>
</tr>
</tbody>
</table>

Figure 2-1. Summary of transit evacuation modeling studies

### 2.4 Evacuation Planning under Uncertainty

Evacuation modeling is subject to uncertainties, and although a great deal of research has been dedicated towards evacuation management, there is a lack of modeling efforts in the literature addressing uncertainty during evacuation. Even after extensive evacuation planning, it is challenging to model evacuations in real world transportation networks due to the associated uncertainties. Public officials face a great challenge during evacuations because of the unpredictability of different aspects of evacuations. For example, studies in the literature show that the total number of people evacuating and their departure times are hardly predictable.
2.4.1 Sources of Uncertainty

There are numerous sources of uncertainties during an evacuation and most of them are not easy to quantify or control. Uncertainties in the evacuation problem can be related to the demand side, to the supply side or to the disaster characteristics. On the demand side, uncertainty can be in terms of total number of evacuees and their locations. Demand uncertainty is associated with human behavior e.g., decision to evacuate or not, timing of evacuation etc. Furthermore, there can be behavioral uncertainties regarding compliance and driving behavior. On the supply side, uncertainty may be caused by impact of disaster on the infrastructure e.g., reduced link capacities, flooded links during hurricanes, limited network connectivity due to collapse of a bridge during earthquakes etc. The limited predictability of the severity of a disaster, e.g. the intensity and track of a hurricane, or the timing of any disaster occurrence, are also some of the factors causing uncertainty during evacuations.

2.4.2 Optimization Under Uncertainty

A large number of problems in production planning and scheduling, location, transportation, finance, and engineering design require that decisions be made in the presence of uncertainty. Problems of optimization under uncertainty are characterized by the necessity of making decisions without knowing what their full effects will be. In addition, a key difficulty in optimization under uncertainty is in dealing with an uncertainty space that is huge and frequently leads to very large-scale optimization models. Approaches to optimization under uncertainty have followed a variety of modeling techniques, and in this section we will review some of the main approaches.
2.4.2.1 Simulation optimization

Simulation based optimization integrates optimization techniques into simulation analysis. Simulation optimization can be defined as the process of finding the optimal decision variable values from among all possibilities without explicitly evaluating each possibility. At each iteration, the simulation-based optimizer provides values for the decision variables, followed by the simulation runs to evaluate the objective and constraints. Simulation optimization is often slow and takes too much time even for a small problem. Figure 2-2 presents a general simulation optimization model (Carson and Maria, 1997). For a more detailed review on simulation based optimization methods, refer Andrad´ottir (1998), Fu (1994), and Fu (2002).

![Simulation optimization model](image)

Figure 2-2. A simulation optimization model

2.4.2.2 Stochastic programming

In stochastic programming, the uncertain elements in a problem can often be modeled as random variables to which the theory of probability can be applied. This approach requires known probability distributions of the uncertain data and it is assumed that the probability structure is unaffected by any of the decisions that are taken during the process (Rockafellar, 2001). The goal of stochastic programming approach is to find some policy which is feasible for all (or almost all) the possible data instances and the expected values is optimized. This approach is the most widely used
approach for optimization under uncertainty and has applications in broad range of areas. We refer the interested reader to several textbooks (Infanger, 1994; Kall and Wallace, 1994; Prékopa, 1995; Birge and Louveaux, 1997) and the references therein for a more comprehensive understanding of stochastic programming.

2.4.2.3 Robust optimization

Stochastic models of decision making under uncertainty assume perfect information and known probability distributions for the random variables. However such precise knowledge of probability distributions is rarely available. Robust optimization is a more recent approach to optimization under uncertainty, in which uncertainty model is not stochastic, but rather set based. In robust optimization, random variables are modeled as uncertain parameters belonging to a bounded set and a decision maker usually protects the system against the worst case value within that set. This notion of modeling uncertainty is useful in many applications and has advantage in computational tractability depending on the problem structure. Interested readers are referred to Ben-Tal and Nemirovski (2002) and Bertsimas et al. (2008) for reviews of the robust optimization methods.

2.4.3 Incorporating Uncertainty in Evacuation Modeling

Due to the associated uncertainties during an evacuation, it is very unlikely that the evacuation process would go (or even close) according to the predetermined plan. There can be significant consequences of uncertainties during real-life mass evacuations and the whole evacuation planning may result in a failure. Past experience has shown that uncertainty might lead to overcrowded shelters, high network congestion and other failure situations. For instance, it was found that the actual number of evacuees during Hurricane Rita from the Galveston and Harris Counties
equaled 1.8 million people, whereas the predicted number was only 686,000 (Lindell and Prater, 2007).

Although the real-life evacuation process shows a significant uncertainty, the general state-of-the-practice ignores the real world uncertainties and assumes fixed demand and capacity. In order to avoid the undesirable surprises during the actual evacuation process, it is essential to develop evacuation plan and decisions which could guarantee a high probability of the actual evacuation to be as close as possible to the predicted one. Therefore, while planning for optimal evacuation measures, it is important to deal with these uncertainties and decisions must be made by accounting for different measures of uncertainties.

In the literature, there are not many studies in evacuation planning focusing on uncertainty. The majority of existing evacuation planning models assumes determinism. Clearly, there is an inconsistency between this deterministic assumption and what is actually experienced in practice. Taking a deterministic approach might lead to sub-optimal evacuation strategies with potentially significant consequences in the real-world.

A few studies have considered uncertainties in evacuation modeling. Yazici and Ozbay (2007) incorporated uncertainty in their Cell Transmission Model (CTM) based System Optimal Dynamic Traffic Assignment (SODTA) model. They approached the problem from stochastic programming perspective and used a chance constraint approach to model the uncertainties in road capacity during evacuation. Changes in clearance time and spatial shelter utilization were analyzed and it was demonstrated that introducing probabilistic link capacities can adjust the overall flow in the network as
well as shelter utilization. However, it was assumed that the total number of evacuees is deterministically known.

Later, Yazici and Ozbay (2010) also included the demand uncertainty in their modeling approach and proposed a SODTA model with probabilistic demand and capacity constraints during evacuation. The problem was modeled based on two different stochastic modeling approaches, namely individual chance constraints and joint chance constraints. It was shown that the model can incorporate different user defined probability values and can be used to calculate the change in clearance time, average travel time and risk exposure measures.

While the work by Yazici and Ozbay is certainly an improvement over previous studies assuming determinism, they adopt the implicit assumption of chance constraint programming that the probability distributions of random demand and capacities are explicitly known. However, this might not be the case because of the very limited or non-existent historical data on evacuations.

Yao et al. (2009) consider evacuation in an uncertain environment and proposed a robust linear programming model considering demand uncertainty. In their study, the demand for evacuation was explicitly considered to be a random variable. They realized the fact that it is not possible to obtain the exact probability distribution of number of evacuees. Road capacities were assumed to be deterministic. It was found that a robust solution improves both feasibility and quality compared to a deterministic solution.

Ng and Waller (2010) presented a CTM-based evacuation route planning model that accounts for both evacuation demand uncertainty as well as road capacity uncertainty. Their model also does not require the assumption that probability
distributions are known explicitly. A novel distribution-free approach is used to provide probabilistic guarantees on the resulting evacuation plan, i.e. they allow for infeasibilities with a pre-specified tolerance level. It was demonstrated that the reliable evacuation plans were able to provide a good estimate of realized evacuation time.

Huibregtse et al. (2010) presented an approach to optimize evacuation measures under uncertainty. The uncertainty in the evacuation problem is related to the evacuees (the demand and the behavior of people) and the hazard (location, time, and intensity). They adopted a scenario-based uncertainty approach (the uncertainty is translated into scenarios which could occur) and defined different criteria to evaluate the effectiveness of evacuation measures. Although the scenario-based approach has some restrictions for real applications, this case study shows the usefulness of dealing with uncertainty in the evacuation problem.

Pel et al. (2010) performed a sensitivity analysis to identify and quantify the impacts of variations in both travel demand and network supply for evacuations. Sensitivity analysis was done using a macroscopic evacuation traffic simulation model EVAQ. The authors vary different aspects such as trip generation, departure rates, route flow rates, road capacities and maximum speeds. It was found that the departure rates and route flow rates have a substantial non-linear impact on the network conditions and arrival pattern, in particular when the network is highly congested, while the trip generation and road capacities have a smaller quasi-linear impact. Figure 2-3 provides a summary of the studies focusing on modeling uncertainty in evacuation planning.
2.5 Contributions of This Dissertation

The review of current trends in emergency planning showed that many of these areas are well studied and that there is a continuous exploration and enhancement of practices and plans. Recently, use of transit has been viewed as an important part of evacuation planning and management. However, there is still a need to address transit evacuation planning to assist transit-dependent residents, in order to develop a comprehensive evacuation response plan.

In the literature, there are a limited number of studies focusing on modeling and optimizing the use of public transit during evacuation. The studies mainly focused on optimizing transit routing plans. In this dissertation, two critical decision making components for transit evacuation planning are studied, i.e., public shelter locations and pick-up locations (staging areas). It has already been shown that the shelter location decisions are extremely important for auto evacuation planning and can greatly
influence the evacuation process. Likewise, public sheltering is an important component of transit evacuation planning and the location of public shelters can be critical to the efficiency of transit evacuation. In addition, all the previous research on modeling transit during evacuation assumes that the pick-up locations (staging areas) for transit are already known, yet this is not the case in real-life evacuations and exact pick-up locations are not known. To our best knowledge, no analytical approach is available in the literature that determines the optimal shelter locations and transit pick-up locations during transit evacuation. In this dissertation, we try to fill this gap.

On the other hand, despite a great deal of research dedicated towards evacuation planning, there is a lack of modeling efforts in the literature addressing uncertainty during evacuation. There is a pressing need for models that incorporate uncertainty and thus assure viable measures for the unanticipated. This dissertation aims to address this important gap.

The main focus of this dissertation is to develop a plan that evacuates those who depend on public transportation to safe areas reliably and as fast as possible during an emergency. More specifically, optimization models are developed for transit-based evacuation planning under demand uncertainty using robust optimization approach, which are more efficient and reliable in the face of demand uncertainty. Demand uncertainty refers to the uncertainty associated with the number of transit-dependent evacuees. In particular, three different models are proposed to model public shelter and pick-up locations for transit evacuation. The resulting plans obtained from the proposed models are expected to perform robustly and effectively in real-world mass evacuations, thereby making the transit-based evacuation planning more reliable and efficient.
CHAPTER 3
ROBUST PUBLIC SHELTER LOCATIONS WITH DEMAND UNCERTAINTY

3.1 Background

The evacuation planning and management has been studied from various aspects, i.e. traffic control strategies, evacuees’ behavior, demand management, shelter and route choice etc. One of the critical tasks in developing an evacuation plan is to determine where evacuees would seek shelter. FDOT (2000) reported poor information on the availability and location of shelters for the evacuees, as one of the weakness of evacuation process during Hurricane Floyd. As indicated by Sherali et al. (1991), the selection of shelter locations from among potential alternatives can greatly influence the evacuation process. However, only a few studies in the literature have explicitly considered the impact of shelter locations on evacuation time. Most of the studies related to shelters during evacuation focus on structural and operational issues rather than location (e.g., ARC, 1992; FEMA, 2000b; Coulbourne et al., 2002 and Yi and Ozdamar, 2007).

Sherali et al. (1991) seemed to be the first to study the shelter location problem for evacuation planning under hurricane/flood conditions. A nonlinear mixed integer programming model was proposed to determine the shelter locations and prescribe paths for evacuees to minimize the system-wide evacuation time. The assumption that the authority has control over evacuees’ shelter and route choice may not be realistic under the disaster conditions. Recognizing this limitation, Kongsomsaksakul et al. (2005) proposed a bi-level programming model to determine the optimal location of shelters during flood evacuation. In their model, the planning authority determines the number and shelter locations (the upper level problem), while the evacuees choose
shelters and routes to evacuate (the lower level problem). The model assumed that the shelter capacities are known and the number of evacuees at each origin is also given. More recently, Ng et al. (2010) proposed a hybrid bi-level model for optimal shelter assignment in emergency evacuations. The upper-level problem specifies a system optimum assignment of the evacuee demand while evacuees choose their routes to the assigned shelters in the lower-level problem. In their study, the shelter locations are assumed to be known.

Above studies indicate the importance of shelter location decision for auto-based evacuation. Likewise, public sheltering is an important component of transit-based evacuation planning and the selection of shelter locations can greatly influence the efficiency of transit operations during evacuation. Especially, for the emergency situation, where the significant portion of demand is transit-dependent and transit can play an important role, it is critical that the shelter location decision be made carefully, identifying the residential location of transit-dependent evacuees in the affected area. However, there is no analytical approach in literature to determine the optimal location of shelters for transit evacuation planning.

In addition, in the planning stage, it is difficult, if not impossible, to estimate accurately the number of transit-dependent evacuees and the public shelter usage during an evacuation. To develop a successful transit evacuation plan, it is critical to proactively consider such a demand uncertainty in determining shelter locations and capacities. Otherwise, the capacities of certain shelters may not be sufficient to accommodate the evacuees while the other shelters remain underutilized. Moreover, travel times to certain shelters may become prohibitively large due to traffic congestion.
Capacity of shelters hereinafter refers to the optimal stocking of supplies at the facilities, and not the physical capacity of shelters.

In developing a mass evacuation plan for transit operations, information regarding the extent to which evacuation plans are coordinated among different counties and neighboring States is essential. The inability to evacuate the transit-dependent population during hurricane Katrina is assumed to be the result of poor coordination and communication of available resources, rather than the lack of transportation resources (USDOT, 2006; Renne 2006). After hurricane Katrina, the U.S. Department of Transportation (USDOT, 2006) stressed the coordination of available resources as a critical issue for a successful mass evacuation plan. However, during large scale evacuation, problem often arises in trying to manage operations involving transit agencies across different counties and jurisdictions. It is observed that the real time coordination between different state and local government while managing an evacuation is very limited. This chapter focuses on transit evacuation planning from a regional level planning perspective for a large-scale evacuation, where transit agencies in different counties are trying to perform their own evacuation and transport the transit-dependent evacuees to the public shelters in order to minimize their evacuation time.

In this chapter a robust model is developed to determine optimal shelter locations and capacities, from a given set of potential sites, under demand uncertainty for transit evacuation. Demand uncertainty refers to the uncertainty associated with the number of transit-dependent evacuees originating from different counties or cities, seeking shelters to evacuate. For the purpose of this dissertation, hereafter, we refer to the use of bus service when mentioning the public transit in the developed models. The proposed
model is formulated as a mathematical program with complementarity constraints, a class of problems difficult to solve. The problem has a bi-level structure. At the upper level, planning authority determines the number and locations of shelters along with their capacities while at the lower level, different transit agencies choose shelters and route to evacuate in order to transport the transit-dependent evacuees. A cutting plane algorithm (Lawphongpanich and Hearn, 2004) is applied to solve the problem.

For the remainder of this chapter, Section 3.2 describes the model components and considerations. Section 3.3 formulates the shelter location model. Section 3.4 proposes an efficient solution algorithm to solve the model. Results from a numerical example based on Sioux Falls network to illustrate the effectiveness of proposed model are presented in Section 3.5. Finally, Section 3.6 provides the summary of this chapter.

3.2 Model Components

3.2.1 Representing Demand Uncertainty

Quite a few techniques have been developed in the literature to accommodate uncertainty in decision making as discussed in Section 2.4.2. Considering that it is practically difficult to obtain the probability distribution of the uncertain demand, a robust optimization approach (e.g. Ben-Tal and Nemirovski, 2002) is adopted in this dissertation to model demand uncertainty. In this approach, the uncertain demand is assumed to be confined to an uncertainty set, and the shelter locations and capacities are determined against the worst-case demand scenario realized in the set.

More specifically, it is assumed that the transit-dependent demand for each county/city can be one of the several possible values whose probabilities of occurrence are unknown. These possible demand values are not difficult to obtain and can be identified, e.g., with alternative forecasts of the magnitude and impact area of the
disaster. For each county \( l \in L \), let \( d^l_s \), \( s = 1, 2, \ldots, S^l \) denote the transit-dependent demand under scenario \( s \), where \( d^l_1 \) corresponds to the nominal demand, i.e., the most-likely forecast. To hedge against uncertainty, a shelter location plan can be designed against a worst-case scenario where each county generates the maximal possible transit-dependent demand. However, this will lead to an overly conservative plan. More specifically, it is rare for all the counties to reach their maximum transit-dependent demand values simultaneously. Designing the location-allocation plan against such a rare case would be overly protective.

With a philosophy that the nature is neither our enemy nor friend, it is assumed that demands for at most \( \Gamma \) counties can deviate from their nominal values simultaneously while the others must remain at their nominal values. \( \Gamma \) is called as the degree of pessimism (also known as uncertainty budget), which is a parameter for controlling the degree of conservatism in the model, reflecting the decision makers’ attitude toward risk. A larger \( \Gamma \) implies that decision makers are more risk averse. This approach of accounting uncertainty (e.g., Bertsimas and Sim, 2003; Lou et al., 2009) leads to computationally-tractable robust formulation and is flexible in controlling the degree of conservatism with respect to decision maker’s attitude towards risk.

Mathematically, let \( z^l_s \) denote a binary variable that indicates whether scenario \( s \) is realized at county \( l \). Since, only one demand scenario can be realized for each county, we have \( \sum_{s=1}^{S^l} z^l_s = 1 \). Also, since at most \( \Gamma \) county demands are allowed to deviate from their nominal values and \( z^l_1 = 1 \) represents that nominal demand value is realized at county \( l \), we have \( \sum_l \sum_{s=2}^{S^l} z^l_s \leq \Gamma \). Additionally, the binary requirement for the variable \( z^l_s \) can be written in the form of a continuous variable by adding constraints, \( z^l_s(1 - z^l_s) = \)
0, 0 \leq z_s^l \leq 1 \text{ which ensures that } z_s^l \text{ is 0 or 1 for each } (l, s). \text{ Thus, the uncertainty set for the transit-dependent demands can be characterized as follows: }

D = \left\{ d \mid d^l = \sum_{s=1}^{S^l} z_s^l d_s^l, \sum_{s=1}^{S^l} z_s^l = 1, \sum_l \sum_{s=2}^{S^l} z_s^l \leq \Gamma, z_s^l (1 - z_s^l) = 0, 0 \leq z_s^l \leq 1 \right\}

\text{where } d \text{ is the demand vector realized for the network and } d^l \text{ is the demand realized for each county } l. \text{ Hereinafter, the superscript } d \text{ denotes variables associated with a particular demand vector } d \in D.

\text{Note that the above uncertainty set does not necessarily include all the possible realizations of transit-dependent demand, which would be really difficult due to limited information at the planning stage. The set is a purely mathematical construct to model demand uncertainty in the robust optimization approach and choice of } D \text{ affects the efficiency and robustness of the resulting robust plan (Ben-Tal and Nemirovski, 2002). Compared with other types of configuration, the above set is easier to obtain because planning agencies are often able to produce a few different demand forecasts as previously discussed.}

\textbf{3.2.2 Notations}

\text{Let } G(N, A) \text{ denote the network of roads, where } N \text{ and } A \text{ are the sets of nodes and links in the network respectively. We denote a link as } a \in A \text{ or the pair of its starting and ending nodes, i.e., } (i, j) \in A. \text{ Assume that the demand during evacuation is originated from a subset of origin nodes } L \subseteq N. \text{ Note that, each origin node here represents a county in the network. Next, let } M \subseteq N \text{ denote a set of potential sites to locate shelters in the network. A binary variable } y_m \text{ represents the shelter location decision, which is 1 if a shelter is located at node } m \in M \text{ and 0 otherwise.}
Let $\Delta$ be the node-link incidence matrix associated with the network and $E^{lm}$ is a vector in $R^{l \times m}$ with two non-zero component: one has a value 1 in the component corresponding to county $l$ and other has a value -1 in the component corresponding to shelter $m$. For a given demand vector $d$, the number of buses between any county and shelter node $q^{lm}_d$ are the results from transit agencies’ destinations choices, and thus are decision variables in the model. The set of feasible county-shelter demand and link flow distributions for the network, denoted as $\varphi(d)$, can be expressed as follows:

$$\varphi(d) = \{ (v^d, x^d, q^d) | v^d = \sum_{l,m} x^{lm}_d, \Delta x^{lm}_d = E^{lm} q^{lm}_d, \sum_m Q^l q^{lm}_d = d^l, \sum_l q^{lm}_d \leq y_mB, x^{lm}_d \geq 0, q^{lm}_d \geq 0 \forall l \in L, m \in M \}$$

where $v$ is the aggregate link flow vector; $x^{lm}$ is the link flow vector (i.e., number of transit vehicles) corresponding to the county-shelter pair $(l,m)$; $q^{lm}$ is the demand in terms of total number of transit vehicles going from county $l$ to shelter $m$; $Q^l$ is the capacity of transit vehicles available with county $l$; $B$ is a sufficiently large number and the inequality, i.e., $\sum_l q^{lm}_d \leq y_mB$, ensures that transit vehicles from each county only go to the sites where a shelter is located. Hereinafter, the subscript or superscript $d$ denotes variables associated with a particular demand vector $d \in D$.

Additionally, it is assumed that there exists background traffic in the network. This background traffic represents the automobile traffic within the network. Let $\bar{v}_a$ denote the estimated background flow on link $a \in A$. Hence, the total flow on link $a$ will be $(v_a + \bar{v}_a)$. Consequently, for calculating the travel time $t_a$ for link $a$, the following form of BPR function is used:

$$t_a = t^0_a \left[ 1 + 0.15 \left( \frac{v_a + \bar{v}_a}{U_a} \right)^4 \right]$$
where $t^0_a$ is the free-flow travel time for link $a$ and $U_a$ is the capacity of link $a$. Since $v_a$ is assumed to be known, we hereinafter denote the travel time function as $t_a(v_a)$.

### 3.2.3 Combined Distribution and Assignment Model

Given the location of shelters, a combined distribution and assignment (CDA) model is used to describe the transit agencies’ choices of which shelters to evacuate and which routes to access the selected shelters. In the CDA model, the network traffic condition is assumed to be in user equilibrium and the selection of shelters is represented by a logit model with a dispersion parameter $\theta$. The CDA model for a specific demand scenario $\hat{d}$ is written as follows:

$$
\min_{v,q} \sum_a \int_0^{v_a} t_a(w) \, dw + \frac{1}{\theta} \sum_l \sum_m q^{lm}(\ln q^{lm} - 1)
$$

s.t.

(3.1) \quad v_a = \sum_{lm} x^{lm}_a \quad \forall \ a \in A

(3.2) \quad \Delta x^{lm} = E^{lm} q^{lm} \quad \forall \ l \in L, m \in M

(3.3) \quad \sum_m Q^l q^{lm} = \hat{d}_l \quad \forall \ l \in L

(3.4) \quad \sum_l q^{lm} \leq y_m B \quad \forall \ m \in M

(3.5) \quad x^{lm}_a \geq 0 \quad \forall \ a \in A, \ l \in L, \ m \in M

(3.6) \quad q^{lm} \geq 0 \quad \forall \ l \in L, \ m \in M

where $v_a$ is the shelter-bound aggregate link flow at link $a$, $x^{lm}_a$ is the shelter-bound flow between the county-shelter pair $(l, m)$ at link $a$, $q^{lm}$ is the total number of transit vehicles going from county $l$ to shelter $m$.

Constraint (3.1) calculates the aggregate link flow. Constraint (3.2) ensures the flow balance between county and shelter nodes in the network. Constraint (3.3) requires
that the total number of evacuees transported from any county $l$ to different shelters $m$
will be equal to the total demand at that particular county $l$. Constraint (3.4) ensures that
transit vehicles only go to the sites where a shelter is located. Constraint (3.5) and (3.6)
are the non-negativity constraints for the link flows and demand.

3.2.4 List of Notations

Below is the list of some useful notations used for the proposed model in this chapter.

$l$ index for origin/county nodes, $l \in L$

$m$ index for potential shelter sites, $m \in M$

$N$ set of nodes

$A$ set of links

$Q^l$ capacity of transit vehicle available at origin/county $l$

$\tilde{c}_m$ fixed cost for establishing a shelter at site $m$

$\hat{c}_m$ unit cost to operate shelter at site $m$

$\theta$ dispersion parameter for CDA model

$T^\text{max}_l$ upper bound on travel time from origin $l$ to any open shelter

$\Gamma$ degree of pessimism

$d$ demand vector

$d^l$ demand realized for origin $l$

$d^l_s$ demand for origin node $l$ under scenario $s$

$q^{lm}$ demand from origin $l$ to shelter $m$

$v_a$ transit flow on link $a$

$\bar{v}_a$ background traffic flow on link $a$
transit link flow corresponding to county-shelter pair \((l, m)\)

travel time for link \(a\)

travel time from origin \(l\) to shelter \(m\)

binary variable indicating which scenario \(s\) is realized for origin \(l\)

binary variable indicating shelter location decision

capacity of shelter \(m\)

Note that, although not all the notations used in this chapter are listed here, they are defined appropriately wherever used.

### 3.3 Model Formulation

In this section, the mathematical formulation for the shelter location problem is presented. For computational efficiency, a static modeling framework is adopted, which appears sufficient for such a macroscopic planning practice. Our future study will examine the impacts of traffic dynamics on the shelter deployment plan. Using the notations defined in previous section, a robust shelter location model or RSL can be formulated as follows:

\[
\text{RSL} \quad \min_{y, K, q, v, x} \sum_{m} y_m \bar{C}_m + \sum_{m} K^m \hat{C}_m
\]

s.t.

\[
\begin{align*}
\sum_{l} Q^l q^l_{d} & \leq K^m & \forall \ m \in M, \ d \in D \\
(v^d, x^d, q^d) & \in \varphi(d) & \forall \ d \in D \\
t_a(v^d_a) + \gamma^d_a & = 0 & \forall \ a \in A, \ d \in D \\
\frac{1}{\theta} (1 + \log q^l_{d}) + Q^l \tau_a^l - \eta^l_{d} \pi^l_{l,d} + \pi^l_{m,a} + \beta^m_{d} & = 0 & \forall \ l \in L, \ m \in M, \ d \in D
\end{align*}
\]
The decision variables in this model include shelter location \( y \), shelter capacity \( K \), county-shelter flow distribution \( q \), link flow distribution variables \( v \) and \( x \), and auxiliary variables \( \gamma^d, \pi^l, \lambda_d^m, \beta_d^m, \eta_d^m \), which are the Lagrangian multipliers for each demand scenario \( d \) associated with Equation (3.1)-(3.6) respectively in the Karush–Kuhn–Tucker (KKT) conditions (e.g., Bazaraa et al., 2006) of the CDA model.

The objective of RSL is to minimize total cost to establish and operate the shelters to accommodate all the potential transit-dependent evacuees. The cost includes a fixed cost for establishing shelters with \( \bar{C}_m \) being the fixed cost for one shelter and a variable cost to operate shelters, which is proportional to the shelter capacity with \( \hat{c}_m \) being the unit cost. Constraint (3.7) ensures that the capacity of each shelter, denoted as \( K \), is always sufficient to accommodate its potential transit-dependent demand under all demand scenarios realized from the uncertainty set \( D \). Constraint (3.8) suggests that the link flow and demand distribution should be feasible. Constraints (3.9)-(3.15) are the KKT conditions for the CDA problem for each demand scenario, ensuring that the distribution of link flow and county-shelter demand is in user equilibrium for the
combined choices of shelters and routes. Constraints (3.16)-(3.17) ensure the shelter location variable to be binary. Finally, constraint (3.18) requires that the travel time for each county to any open shelter, $t_{dn}^m$, is less than an upper bound $T_{l}^{max}$. This requirement is to ensure transit-dependent evacuees from all the counties to reach a safe location within a specified time. Note that since a logit model is used to describe shelter choices, an open shelter will be utilized by transit agencies from all counties, even though the number from certain county could be very small. This is due to the fact that a logit model will always lead to a strictly positive probability even for the least attractive option.

As formulated, RSL is a mathematical program with complementarity constraints (MPCC), a class of problems difficult to solve (e.g., Scheel and Scholtes, 2000). The formulation has four sets of complementarity constraints; three sets to ensure that the flow and demand distribution is in user equilibrium and the other one to make binary variables $y_m$ continuous. Standard stationarity conditions such as the KKT conditions do not hold for MPCC (Scheel and Scholtes, 2000) and many (e.g., Luo et al., 1996, and references cited therein) have proposed special algorithms to solve them. However, many of these algorithms work well for small and medium problems and others, especially those based on solving equivalent nonlinear programs (e.g., Fletcher and Leyffer, 2004), can handle larger problems.

### 3.4 Solution Algorithm

#### 3.4.1 Solving Robust Shelter Location Problem

In this section a cutting plane algorithm (Lawphongpanich and Hearn, 2004) to solve RSL is presented, considering that the number of demand vectors in the uncertainty set $D$ can be extremely large. The algorithm solves a sequence of restricted
RSL problems with respect to a subset of demand vectors \( \bar{D} \subset D \) and then solves another problem to generate demand vectors one at a time as needed to expand \( \bar{D} \). The restricted RSL problem (R-RSL) is written as follows:

\[
\text{R-RSL}
\]

\[
\min_{\gamma, \lambda, \eta, \pi, \tau} \sum_{m} y_m \hat{c}_m + \sum_{m} K^m \tilde{c}_m
\]

s.t.

\[
\sum_{l} Q^l q^l_{dm} \leq K^m \quad \forall \ m \in M, \ d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.19)
\]

\[
(\nu^d, x^d, q^d) \in \varphi(d) \quad \forall \ d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.20)
\]

\[
t_a(v^d) + \gamma^d = 0 \quad \forall \ a \in A, \ d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.21)
\]

\[
\frac{1}{\theta} (1 + \log q^m_d) + Q^l \tau^l_d - \eta^l_d - \pi^l_{i,d} + \pi^l_{m,d} + \beta^m_d = 0 \forall \ l, m, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.22)
\]

\[
-\gamma^a_d - \lambda^l_{a,d} + \pi^l_{i,d} - \pi^l_{j,d} = 0 \forall \ (i, j) \in A, \ l \in L, m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.23)
\]

\[
\beta^m_d (\sum_{l} q^l_{dm} - y_mB) = 0 \quad \forall \ m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.24)
\]

\[
\lambda^l_{d} x^l_{dm} = 0 \quad \forall \ l \in L, m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.25)
\]

\[
\eta^l_{d} q^l_{dm} = 0 \quad \forall \ l \in L, m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.26)
\]

\[
\eta^l_{d}, \lambda^l_{d}, \beta^m_d \geq 0 \quad \forall \ l \in L, m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.27)
\]

\[
t^l_{dm} \leq T^l_{\text{max}} \quad \forall \ l \in L, m \in M, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (3.28)
\]

and (3.16)-(3.17)

Let \((\bar{y}, \bar{R})\) solve the R-RSL globally and let \(\bar{M} \subseteq M\) denote the corresponding set of locations with shelters. Then, the solution \((\bar{y}, \bar{R})\) also solves the original RSL problem if the shelter capacity \(\bar{R}\) is sufficient and travel time between each county \(l \in L\) and open shelter \(\bar{m} \in \bar{M}\) is within its threshold value, under all demand scenarios in \(D\). For a given
location of shelters $\bar{m}$ and demand vector $d$, the set of all feasible link flow and county-shelter distributions for the network, denoted as $\bar{\varphi}(d)$, is expressed as follows:

$$\bar{\varphi}(d) = \{(v^d, x^d, q^d) | v^d = \sum_{lm} x_{lm}^d, \Delta x_{lm}^d = E_{lm} q_{lm}^d, \sum_{m} Q_l^l q_{lm}^d = d^l, x_{lm}^d \geq 0, q_{lm}^d \geq 0 \ \forall l, \bar{m}\}$$

To verify this mathematically, following worst-case demand problem (WCD) is solved:

$$\text{WCD}$$

$$\max_{x,d} \left[ \sum_{m} \max \left( \sum_{l} Q_l q_{lm}^d - \bar{R}_m, 0 \right) + \sum_{l} \sum_{m} \max(t_{lm} - T_{lm}^l, 0) \right]$$

s.t.

$$ (v^d, x^d, q^d) \in \bar{\varphi}(d) \quad \forall d \in D \quad (3.29)$$

and (3.9)-(3.15)

The objective of WCD is to find a demand vector $d$ to create the most “trouble” for the current design of shelters. More specifically, the demand scenario will make, to a largest possible extent, the shelter capacity insufficient and travel time larger than the required threshold. If the optimal objective value of WCD problem is no greater than zero, then $(\bar{y}, \bar{R})$ is an optimal solution to the RSL problem. Otherwise, an improved solution can be obtained by expanding the demand vector set $\bar{D} \cup \{d^w\}$, where $d^w$ is the solution to above WCD.

Because the number of demand vectors in the uncertainty set $D$ is finite, the cutting plane algorithm terminates after a finite number of iterations. Below is the formal solution procedure of the cutting plane algorithm outlined above.

Cutting Plane Algorithm:

Step 0: Choose an initial demand vector $d_1 \in D$. Set $n = 1$ and $\bar{D} = \{d_1\}$. 

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Step 1: Solve the R-RSL problem with a subset of demand vectors $\bar{D}$ (Section 3.4.2) and let $(\hat{y}^n, \hat{R}^n)$ denote the resulting optimal solution.

Step 2: Solve the WCD problem associated with the shelter location $\hat{y}^n$ and shelter capacities $\hat{R}^n$ (Section 3.4.3) and let $d_n$ denote the optimal solution and $\hat{W}$ denote the corresponding objective value.

Step 3: If $\hat{W} \leq \epsilon$, stop and $(\hat{y}^n, \hat{R}^n)$ is an optimal solution to the RSL problem. Otherwise, set $\bar{D} = \bar{D} \cup \{d_n\}$, $n = n + 1$ and go to step 1.

The solution obtained from the above cutting plane algorithm will be global optimal to the RSL problem, if the R-RSL and WCD problems can be solved optimally. However, both the sub problems (R-RSL and WCD) are still MPCC, a class of non-convex problems difficult to solve to the global optimality.

### 3.4.2 Solving Restricted Robust Shelter Location Problem

An active set algorithm (Zhang et al., 2009) is applied to solve the R-RSL. The basic idea of the algorithm is to solve a sequence of relaxation of the original MPCC problem and strategically updating the feasible design (shelter locations in our case) using the dual information associated with the relaxed problem.

Given a feasible shelter locations $\bar{y}$ to R-RSL, consider the two index sets defined as $\Omega_0 = \{m|\bar{y}_m = 0\}$ and $\Omega_1 = \{m|\bar{y}_m = 1\}$. The index-set-based relaxed R-RSL problem (R-RSL-I) can thus be written as follows:

\[
\min_{\gamma_{m,q,v,x}, \lambda_{y,\eta,\pi,r}} \sum_{m} \gamma_m \tilde{c}_m + \sum_{m} K^m \tilde{c}_m \\
\text{s.t.} \\
\gamma_m = 0 \quad \forall \ m \in \Omega_0 \\
\]

(3.30)
The solution to R-RSL-I is very easy to obtain without directly solving the problem. For a known feasible shelter locations \( \bar{y} \), the flow and county-shelter demand distribution can be easily obtained by solving the CDA model (Section 3.2.2) for each demand scenario in \( \bar{D} \). With the known O-D demand distribution for each demand scenario, the minimum shelter capacities and total cost can be easily calculated. With the optimal solution to R-RSL-I as the initial solution, the problem is solved directly once again. The purpose is to obtain the Lagrangian multipliers associated with the constraints (3.30) and (3.31), which provide valuable information on the marginal impact of adjusting the index sets to obtain an updated shelter location plan.

Let \( \mu_m \) and \( \nu_m \) denote the Lagrangian multipliers associated with (3.30) and (3.31) respectively. To update the index sets, the following knapsack problem is considered:

\[
\text{min}_{y,p} \sum_{m \in \Omega_0} \mu_m p_m - \sum_{m \in \Omega_1} \nu_m p_m
\]

s.t.
\[
\begin{align*}
y_m &= p_m & \forall m \in \Omega_0 & \quad (3.32) \\
y_m &= 1 - p_m & \forall m \in \Omega_1 & \quad (3.33) \\
\sum_{m \in \Omega_0} \mu_m p_m - \sum_{m \in \Omega_1} \nu_m p_m & \geq \psi & \quad (3.34) \\
\sum_m y_m & \geq 1 & \quad (3.35) \\
p_m & \text{is binary} & \forall m
\end{align*}
\]

where the binary variable \( p_m \) indicates whether to flip the value of \( y_m \) or not. Constraints (3.32) and (3.33) strategically update the value of \( y_m \) based on the value of \( p_m \). For
example, if \( y_m = 0 \) (i.e. \( m \in \Omega_0 \)), and \( y_m \) is flipped (i.e. \( p_m = 1 \)) then the updated value of \( y_m \) will become 1, otherwise the updated value of \( y_m \) will remain 0 if \( y_m \) is not flipped (i.e. \( p_m = 0 \)). Thus, the objective of the above knapsack problem is to minimize the estimated change to the total cost by adjusting the current shelter locations and a negative objective value indicates that a reduction in total cost is possible. Constraint (3.34) is introduced to help find suboptimal solutions. Initially, \( \psi = -\infty \) and solving KP-RSL would yield updated shelter locations, with the most negative objective value \( \alpha \). If the updated shelter location actually yields a lower total estimated cost (solving the R-RSL-I with updated locations to calculate the new total cost and comparing with the previous value of total estimated cost) and is feasible (i.e. satisfying the travel time requirement), then an improved shelter location design is found. However, because the Lagrangian multipliers are only estimate of marginal changes while the actual changes are either zero or one, the new design may not lead to an actual decrease in the total cost. If the new shelter location is not a better and feasible solution, then another design is generated by setting \( \psi = \alpha + \delta_{RSL} \), where \( \delta_{RSL} \) is a sufficiently small positive number. Constraint (3.35) ensures that there is at least one open shelter in the updated shelter location plan. If the optimal objective value of KP-RSL is zero, then no adjustment in the shelter locations leads to a better solution and the algorithm is terminated. The procedure to solve R-RSL can be described as follows:

Active Set Algorithm (R-RSL):

Step 0: Locate shelters at all potential sites and set it as an initial feasible shelter location plan \( \bar{y} \).
Step 1: Solve the R-RSL-I problem with \( \bar{y} \) and let \( \bar{\xi} \) denote the resulting optimal objective value. Obtain the Lagrangian multipliers \( \mu_m \) and \( \nu_m \). Set \( \psi = -\infty \).

Step 2: Solve the KP-RSL problem. Let \( (y, p) \) denote the resulting optimal solution and \( \alpha \) denote the corresponding objective value. If \( \alpha \geq 0 \), stop and \( \bar{y} \) is the optimal solution to R-RSL. Otherwise go to step 3.

Step 3: Solve the R-RSL-I with new updated solution \( y \). Let \( \xi \) denote the corresponding objective value. If \( \xi < \bar{\xi} \) and the new updated solution \( y \) is feasible then an improved shelter location design is found, set \( \bar{y} = y \), update the active sets \( \Omega_0 \) and \( \Omega_1 \) and go to Step 1. Otherwise, set \( \psi = \alpha + \delta_{RSL} \) and go to Step 2.

### 3.4.3 Solving Worst Case Demand Problem

WCD is also an MPCC problem with a structure similar to R-RSL and therefore the same active set algorithm is applied to solve it. Given a demand vector \( \bar{d} \) and its corresponding \( \bar{z} \), consider two index sets defined as \( \Psi_0 = \{(l, s) | \bar{z}_s^l = 0\} \) and \( \Psi_1 = \{(l, s) | \bar{z}_s^l = 1\} \). The index-set-based relaxed WCD problem (WCD-I) can thus be written as follows:

\[
\begin{align*}
\text{WCD-I} \\
\max_{z, d} \left[ \sum_m \max \left( \sum_l Q_l^t q_m^l - \bar{R}_m, 0 \right) + \sum_m \sum_l \max(t_m^l - T_l^{\max}, 0) \right]
\end{align*}
\]

s.t.
\[
\begin{align*}
z_s^l &= 0 \quad \forall \ (l, s) \in \Psi_0 \\
z_s^l &= 1 \quad \forall \ (l, s) \in \Psi_1 \\
d^l &= \sum_{s=1}^S z_s^l d_s^l \quad \forall \ l \in L \\
\sum_{s=1}^S z_s^l &= 1 \quad \forall \ l \in L
\end{align*}
\]
\begin{align}
\sum_l \sum_{s=2}^{S^l} z^l_s & \leq \Gamma \quad \forall l \in L \\
(v^d, x^d, q^d) & \in \bar{\varphi}(d) \\
t_a(v^d_a) + y^d_a & = 0 \quad \forall a \in A \\
\frac{1}{\theta} \left(1 + \log q^l_m \right) + Q^l \tau^l_d - \eta^l_d - \pi^l_{i,d} + \pi^l_{m,a} + \beta^m_d & = 0 \quad \forall l \in L, m \in \overline{M} \\
-\gamma^a_d - \lambda^l_{a,d} + \pi^l_{i,d} - \pi^l_{j,d} & = 0 \quad \forall (i,j) \in A, \ l \in L, m \in \overline{M} \\
\beta^m_d (\sum_l q^l_d - y_mB) & = 0 \quad \forall m \in \overline{M} \\
\lambda^l_d x^l_d & = 0 \quad \forall l \in L, m \in \overline{M} \\
\eta^l_d q^l_d & = 0 \quad \forall l \in L, m \in \overline{M} \\
\eta^l_d, \lambda^l_d, \beta^m_d & \geq 0 \quad \forall l \in L, m \in \overline{M}
\end{align}

Similar to R-RSL-I, the optimal solution to WCD-I is also easy to obtain. For a given demand vector \(\bar{d}\), the corresponding O-D and flow distribution can be easily obtained by solving the CDA model. In a similar manner, WCD-I is then solved directly using the optimal solution as an initial solution. Let \(\sigma^l_s\) and \(\rho^l_s\) denote the Lagrangian multipliers associated with (3.36) and (3.37) respectively. To update the index sets, the following knapsack problem is considered:

\[\text{KP-WCD}\]

\[
\max_{d,z,r} \sum_{(l,s)} \psi_0 \sigma^l_s r^l_s - \sum_{(l,s)} \psi_1 \rho^l_s r^l_s
\]

\[\text{s.t.}\]

\[
z^l_s = r^l_s \quad \forall (l,s) \in \Psi_0
\]

\[
z^l_s = 1 - r^l_s \quad \forall (l,s) \in \Psi_1
\]

\[
\sum_{(l,s) \in \Psi_0} \sigma^l_s r^l_s - \sum_{(l,s) \in \Psi_1} \rho^l_s r^l_s \leq \omega
\]
\( r_s^l \) is binary \( \forall l, s \)

and (3.38)-(3.40)

where binary variable \( r_s^l \) indicates whether to flip the value of \( \tilde{z}_s^l \) or not.

The objective of the above knapsack problem is to maximize the estimated change to the worst-case objective value by adjusting \( \tilde{z} \) and a positive objective value indicates that an increase in worst-case objective is possible. The procedure to solve WCD can be described as follows:

Active Set Algorithm (WCD):

Step 0: Choose an initial demand vector \((\tilde{d}, \tilde{z})\).

Step 1: Solve the WCD-I problem with \((\tilde{d}, \tilde{z})\) and let \( \bar{W} \) denote the resulting worst-case objective value. Obtain the Lagrangian multipliers \( \sigma_s^l \) and \( \rho_s^l \). Set \( \omega = \infty \).

Step 2: Solve the KP-WCD problem. Let \((d, z, r)\) denote the resulting optimal solution and \( \kappa \) denote the corresponding objective value. If \( \kappa \leq 0 \), stop and \( \tilde{d} \) is the worst-case demand to WCD. Otherwise go to step 3.

Step 3: Solve the WCD-I with the new updated solution \((d, z)\). Let \( W \) denote the corresponding objective value. If \( W > \bar{W} \) then a worse demand vector is found, set \( \bar{d} = d \), update the active sets \( \Psi_0 \) and \( \Psi_1 \) and go to Step 1. Otherwise, set \( \omega = \kappa - \delta_{\text{WCD}} \) and go to Step 2.

3.5 Numerical Example

This section presents results from solving the robust shelter location model for the Sioux Falls network (LeBlanc et al., 1975). The cutting plane algorithm is implemented in GAMS (Brooke et al., 1992) in which CDA, R-RSL-I and WCD-I problems are solved
using CONOPT (Drud, 1992) and the knapsack problems (KP-RSL and KP-WCD) using CPLEX (CPLEX, 2008).

The Sioux Falls network contains 76 links and 24 nodes. Mass evacuation is considered because of a hypothetical disaster in the region. Eight potential sites to locate shelter in the network are assumed as nodes 1, 4, 8, 13, 14, 18, 20, and 22. The remaining 16 nodes are the origin nodes with nominal demands as shown in Table 3-1. Each origin node which represents a county is assumed to have three demand scenarios. The other two demand scenarios, low and high, are generated by multiplying the nominal demand by a uniformly-distributed number between (0.5, 1) and (1.5, 2). The demand values reported here are in terms of number of transit vehicles after considering the total number of transit-dependent evacuees and the capacity of available buses.
Table 3-1. Demand scenarios at each county

<table>
<thead>
<tr>
<th>County</th>
<th>Nominal Demand</th>
<th>Low Demand</th>
<th>High Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 2</td>
<td>4.00</td>
<td>2.34</td>
<td>6.32</td>
</tr>
<tr>
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<td>2.76</td>
<td>4.88</td>
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<td>3.88</td>
<td>9.17</td>
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<td>7.00</td>
<td>4.55</td>
<td>12.02</td>
</tr>
<tr>
<td>Node 7</td>
<td>11.00</td>
<td>7.11</td>
<td>18.48</td>
</tr>
<tr>
<td>Node 9</td>
<td>14.00</td>
<td>8.57</td>
<td>23.46</td>
</tr>
<tr>
<td>Node 10</td>
<td>30.00</td>
<td>20.25</td>
<td>46.97</td>
</tr>
<tr>
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<td>20.00</td>
<td>18.56</td>
<td>31.50</td>
</tr>
<tr>
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<td>12.00</td>
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<td>21.53</td>
</tr>
<tr>
<td>Node 15</td>
<td>16.00</td>
<td>12.00</td>
<td>30.65</td>
</tr>
<tr>
<td>Node 16</td>
<td>22.00</td>
<td>21.98</td>
<td>35.54</td>
</tr>
<tr>
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</tr>
<tr>
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<td>9.96</td>
<td>18.88</td>
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<td>8.00</td>
<td>7.05</td>
<td>13.21</td>
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<tr>
<td>Node 23</td>
<td>13.00</td>
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<td>20.22</td>
</tr>
<tr>
<td>Node 24</td>
<td>7.00</td>
<td>5.74</td>
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<td>338.08</td>
</tr>
</tbody>
</table>

The locations of origins (counties) and potential shelter nodes are shown in Figure 3-1. The original data for each link in LeBlanc et al. (1975) consist of two parameters and they are transformed into the link’s free-flow travel time and capacity. As shown in Table 3-2, the threshold of maximum travel time from each origin to any shelter is set to be slightly greater than the maximum travel time from the origin when all origins have the highest demand simultaneously and shelters are located at all potential sites, i.e., \( y_m = 1 \ \forall m \). This is to ensure that at least a feasible shelter location plan exists.
Figure 3-1. Sioux Falls network with origin nodes and potential shelter sites
Table 3-2. Maximum travel time for each county to shelters

<table>
<thead>
<tr>
<th>County</th>
<th>Maximum Travel Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 2</td>
<td>44</td>
</tr>
<tr>
<td>Node 3</td>
<td>36</td>
</tr>
<tr>
<td>Node 5</td>
<td>48</td>
</tr>
<tr>
<td>Node 6</td>
<td>52</td>
</tr>
<tr>
<td>Node 7</td>
<td>38</td>
</tr>
<tr>
<td>Node 9</td>
<td>90</td>
</tr>
<tr>
<td>Node 10</td>
<td>90</td>
</tr>
<tr>
<td>Node 11</td>
<td>92</td>
</tr>
<tr>
<td>Node 12</td>
<td>36</td>
</tr>
<tr>
<td>Node 15</td>
<td>95</td>
</tr>
<tr>
<td>Node 16</td>
<td>65</td>
</tr>
<tr>
<td>Node 17</td>
<td>100</td>
</tr>
<tr>
<td>Node 19</td>
<td>100</td>
</tr>
<tr>
<td>Node 21</td>
<td>36</td>
</tr>
<tr>
<td>Node 23</td>
<td>36</td>
</tr>
<tr>
<td>Node 24</td>
<td>36</td>
</tr>
</tbody>
</table>

For simplification, it is assumed that the background traffic in the network is zero. The dispersion parameter in the CDA model is assumed to be 0.1. The fixed costs $\bar{c}_m$, associated with establishing shelters at potential sites are shown in Table 3-3 and the variable cost to operate shelters is assumed to be 100 per unit capacity. It is further assumed that capacity of all the buses available in different counties is same and is assumed to be 30.
Table 3-3. Fixed cost associated with establishing the shelters at potential sites

<table>
<thead>
<tr>
<th>Potential Shelter Sites</th>
<th>Fixed Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 1</td>
<td>4100</td>
</tr>
<tr>
<td>Node 4</td>
<td>4200</td>
</tr>
<tr>
<td>Node 8</td>
<td>4600</td>
</tr>
<tr>
<td>Node 13</td>
<td>4200</td>
</tr>
<tr>
<td>Node 14</td>
<td>4400</td>
</tr>
<tr>
<td>Node 18</td>
<td>4500</td>
</tr>
<tr>
<td>Node 20</td>
<td>4100</td>
</tr>
<tr>
<td>Node 22</td>
<td>4300</td>
</tr>
</tbody>
</table>

Table 3-4 compares the results for the nominal, robust and worst-case plans for the optimal shelter plans in the network. Robust plan is obtained using the degree of pessimism, $\Gamma = 5$ while the nominal plan is obtained with $\Gamma = 0$ (deterministic demands with nominal value). The worst-case plan is the optimal plan against deterministic demand with all highest values. It is equivalent, in this particular example, to solve the robust formulation with the degree of pessimism, $\Gamma = 16$.

Table 3-4. Comparison of nominal, robust and worst-case plans

<table>
<thead>
<tr>
<th>Shelter Locations</th>
<th>Capacity</th>
<th>Shelter Locations</th>
<th>Capacity</th>
<th>Shelter Locations</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 4</td>
<td>2098.8</td>
<td>Node 4</td>
<td>2400.9</td>
<td>Node 1</td>
<td>900.9</td>
</tr>
<tr>
<td>Node 18</td>
<td>2258.1</td>
<td>Node 14</td>
<td>1217.7</td>
<td>Node 4</td>
<td>1432.2</td>
</tr>
<tr>
<td>Node 22</td>
<td>1643.1</td>
<td>Node 18</td>
<td>2852.1</td>
<td>Node 8</td>
<td>1265.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Node 22</td>
<td>1730.4</td>
<td>Node 13</td>
<td>1059.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Node 14</td>
<td>1270.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Node 18</td>
<td>2423.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Node 22</td>
<td>1791.3</td>
</tr>
</tbody>
</table>

| Total Capacity Estimate | 6000.00 | 8201.10 | 10143.00 |
| Total Cost Estimate     | 613,000.00 | 837,510.00 | 1,044,600.00 |
| Reliability of Shelter Capacity Provision | 4.20 % | 90.40 % | 100.00 % |
| Probability of Meeting Travel Time Requirement | 90.52 % | 98.82 % | 100.00 % |
In order to evaluate the performance of these three different plans, travel demand vectors are randomly generated by assuming that there is an equal probability of occurrence of the nominal, low and high demand scenarios at each origin. For each sampled demand vector, the CDA model is solved to evaluate whether the shelter capacities are sufficient and the travel times to shelters are less than the thresholds under three different plans. The percent times that these two requirements are met are reported in Table 3-4. Note that a sufficient large number of demand vectors are drawn such that the variance of the estimate of the reliability of shelter capacity provision or the reliability of meeting travel time requirement is less than 0.1.

Difference in the total capacity and cost estimates for the nominal, robust and worst-case plan can be observed from Table 3-4. The worst-case plan achieves 100% reliability in the capacity provision and meeting the travel time requirement. However, it is expensive and may be too conservative. On the other hand, the robust plan, saving 20% in the total cost, achieves nearly 90% and 99% chance in providing sufficient capacity and meeting the travel time requirements. The nominal plan saves 42% in the total cost, but achieves only 4% reliability of sufficient capacity provision. This underscores the importance of accommodating the demand uncertainty in evacuation planning.

Table 3-5 compares the nominal and worst-case plans against the robust plans with varying the degree of pessimism. In general, as the degree of pessimism increases, the capacity provision and travel time guarantee becomes more and more reliable while the total cost increases. It can be seen that robust plans with a large
enough degree of pessimism e.g., $\Gamma \geq 5$ provides a good cost saving and a superior performance with the reliability of 90% or more.

Note that the degree of pessimism reflects the risk-taking attitude of the decision maker. However, without doing a posterior simulation as above, it is difficult to decide the appropriate value for the degree of pessimism. It is recommended to start with a half of the number of the origins with uncertain demand, i.e., $\Gamma = 8$ for the numerical example. Our future research is to examine how to construct an uncertainty set to explicitly ensure the performance of the resulting robust plan.

Table 3-5. Reliability of cost estimate and travel time constraint with varying degree of pessimism

<table>
<thead>
<tr>
<th>Degree of Pessimism ($\Gamma$)</th>
<th>Total Cost Estimate</th>
<th>Savings in Cost (Compared to the Worst-Case Plan)</th>
<th>Total Shelter Capacity Estimate</th>
<th>Reliability of Shelter Capacity Provision</th>
<th>Probability of Meeting Travel Time Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (nominal case)</td>
<td>613,000.00</td>
<td>41.3 %</td>
<td>6000.00</td>
<td>4.17 %</td>
<td>90.52 %</td>
</tr>
<tr>
<td>1</td>
<td>672,970.10</td>
<td>35.6 %</td>
<td>6599.70</td>
<td>23.07 %</td>
<td>94.78 %</td>
</tr>
<tr>
<td>2</td>
<td>726,539.50</td>
<td>30.4 %</td>
<td>7094.40</td>
<td>42.13 %</td>
<td>97.04 %</td>
</tr>
<tr>
<td>3</td>
<td>771,420.00</td>
<td>26.2 %</td>
<td>7543.20</td>
<td>64.37 %</td>
<td>97.25 %</td>
</tr>
<tr>
<td>4</td>
<td>808,109.80</td>
<td>22.6 %</td>
<td>7907.10</td>
<td>82.38 %</td>
<td>98.37 %</td>
</tr>
<tr>
<td>5</td>
<td>837,510.20</td>
<td>19.8 %</td>
<td>8201.10</td>
<td>90.40 %</td>
<td>98.82 %</td>
</tr>
<tr>
<td>6</td>
<td>880,099.90</td>
<td>15.7 %</td>
<td>8586.00</td>
<td>92.15 %</td>
<td>99.87 %</td>
</tr>
<tr>
<td>7</td>
<td>926,329.70</td>
<td>11.3 %</td>
<td>9048.30</td>
<td>97.22 %</td>
<td>99.92 %</td>
</tr>
<tr>
<td>8</td>
<td>938,360.50</td>
<td>10.2 %</td>
<td>9168.60</td>
<td>98.18 %</td>
<td>100.00 %</td>
</tr>
<tr>
<td>9</td>
<td>955,529.70</td>
<td>8.5 %</td>
<td>9294.30</td>
<td>98.63 %</td>
<td>100.00 %</td>
</tr>
<tr>
<td>10</td>
<td>975,390.00</td>
<td>6.6 %</td>
<td>9492.90</td>
<td>98.75 %</td>
<td>100.00 %</td>
</tr>
<tr>
<td>16 (worst-case)</td>
<td>1,044,600.00</td>
<td>-</td>
<td>10143.00</td>
<td>100.00 %</td>
<td>100.00 %</td>
</tr>
</tbody>
</table>
3.6 Summary

The chapter focuses on regional level transit evacuation planning and develops robust shelter location model to determine optimal shelter locations and their capacities under demand uncertainty. The proposed model is formulated as a mathematical program with complementarity constraints, in which the planning authority determines the shelter locations along with their capacities, while the transit agencies in different counties choose the shelters and routes to evacuate. A cutting plane algorithm was proposed to solve the problem.

Numerical results from the Sioux Falls network verify the importance of considering demand uncertainty and using a robust approach for evacuation planning. The worst-case plan can be 100% reliable in satisfying the transit-dependent demand and meeting the travel time requirement under all demand scenarios, but is much more expensive and very conservative. On the other hand, a nominal plan may be less expensive, but is much less reliable too. In contrast, the robust plan can be significantly economical and produces nearly the same level of performance. As the degree of pessimism increases, the robust plans become more reliable and expensive as well. Depending on decision makers’ attitude towards risk, an appropriate value of the degree of pessimism can be chosen.

Although only demand uncertainty is incorporated in this model, the proposed model framework is general enough to accommodate other types of uncertainty, such as capacity uncertainty. As part of future work, we plan to implement the proposed model and solution algorithm for larger-scale networks. Regional coordination is an important aspect of mass evacuation transit planning. Consequently, it is important to create coordinated transit evacuation plan to evacuate transit-dependent residents more
effectively. In future, the proposed model could be extended to incorporate the coordination among the transit agencies in different counties.
CHAPTER 4
TRANSIT PICK-UP LOCATIONS AND BUS ALLCOATION WITH DEMAND UNCERTAINTY

4.1 Background

This chapter focuses on transit evacuation planning on a local level, i.e., from the perspective of a transit agency in a particular county or city. In the previous chapter, we discussed that the public shelter location as a critical component for transit evacuation planning. Another critical component of transit evacuation planning is the location of transit pick-up points, also known as staging areas, within the affected area. Transit pick-up points refer to the locations where transit-dependent evacuees will assemble and the transit agencies will allocate the available transit vehicles to pick up those evacuees and transport them to public shelters.

All the previous studies in the literature on modeling the transit-based evacuation (Chapter 2) typically assume that the pick-up locations for transit are always known and given, but it is not the case with real-life evacuations. Renne et al. (2008) found that out of 26 studied cities that had published their evacuation plans, 20 cities had some provision for transit-dependent residents. However, most of the plans did not specify any specific locations for transit pick-up points. Evacuation plans for ten cities mentioned pick-up points during an evacuation but did not specify any location, two plans advise residents to wait on main roads, four advise residents to wait along bus routes and only four (New York, Boston, Cleveland, Charlotte) show maps of exact pick-up locations. Optimal locations of pick-up points can influence the total evacuation time and the efficiency of the evaluation plans. However, no analytical model has been proposed to help local governments and transit agencies with this decision making. This chapter aims to fill this void.
This chapter presents a robust model for determining the optimal pick-up locations and bus allocation for transit-based evacuation planning with demand uncertainty. More specifically, for transit-based evacuation planning, optimal locations where evacuees’ will assemble are determined, and allocate transit vehicles to assembly locations to pick up the evacuees and transport them to public shelters. The location of public shelters is assumed to be known and given. We consider that the number of transit-dependent evacuees is uncertain and attempt to design a location-allocation plan that performs reasonably well in any realization of the uncertain demand. The proposed model is formulated as an integer program and is solved via a cutting-plane algorithm.

In Section 4.2, modeling considerations are discussed in detail. Section 4.3 then formulates the pick-up location model, which is solved using a cutting-plane scheme proposed in Section 4.4. Results from a numerical example based on Sioux Falls network to illustrate the effectiveness of proposed model are then presented in Section 4.5. Summary of this chapter is provided in Section 4.6.

4.2 Modeling Considerations

4.2.1 Basic Assumptions

Consider a situation when residents are required to evacuate in the event of a disaster. In response, many residents will use their own private vehicles to evacuate while transit-dependent residents will rely on the public transportation system to take them to safe locations. When the evacuation orders are released, it is assumed that transit-dependent evacuees from different residential locations will assemble at designated pick-up locations, and allocated transit vehicles will pick up and transport them to public shelters. In this model, we determine the optimal locations of pick-up
points and allocation of a given number of transit vehicles to pick up evacuees in order to minimize the total evacuation time.

In our modeling framework, it is assumed that evacuees will go to their nearest pick-up points. It is further assumed that evacuees will accumulate at the pick-up locations before the start of the evacuation process and the proposed model thus does not take into account delays or times for evacuees to get to the pick-up locations.

The number of available transit vehicles and their capacities are assumed to be known in the model. For allocation, each vehicle is assigned to a particular pick-up location. The vehicle can make trips to different public shelters, but it only serves its assigned one pick-up point without going to another one. Locations of open public shelters and their capacities available for transit-dependent evacuees are considered to be known. It is further assumed that transit vehicles, in a limited number, have negligible impact on overall congestion, and thus the travel time between pick-up points and public shelters are given and constant.

Evacuation planning is multifaceted with a variety of objectives. For this model, the location-allocation plan is optimized to minimize the maximum total evacuation time realized in the uncertainty set. Other objectives can be used, such as minimizing the longest evacuation time, maximizing the total number of people evacuated, minimizing the total number of vehicles needed. According to a survey report (Zhang and He, 2008), the highest priority for transit evacuation planning is often to minimize the total evacuation time of all the transit-dependent evacuees.
4.2.2 Demand Uncertainty

Numbers of transit-dependent evacuees at different residential locations are considered to be uncertain, and an optimal location-allocation plan is designed against the uncertain demands.

A similar approach as discussed in Section 3.2.1 is used to model demand uncertainty for transit-dependent evacuees. Let \( d_s^i \), \( s = 1,2, \ldots, S \) denote the possible demand value at the resident location or demand point \( i \in I \), with \( d_1^i \) corresponding to the nominal demand value, i.e., the most-likely forecast. Let \( z_s^i \) be a binary variable that indicates whether demand value \( s \) is realized at demand point \( i \). Since, only one demand value can be realized at each demand point, we have \( \sum_{s=1}^{S} z_s^i = 1 \). Also, since at most \( \Gamma \) demand points are allowed to deviate from their nominal values and \( z_1^i = 1 \) represents that the nominal demand value is realized at demand point \( i \), we have \( \sum_i \sum_{s=2}^{S} z_s^i \leq \Gamma \). Thus, the uncertainty set for the transit-dependent evacuation demand at demand points can be characterized as follows:

\[
D = \left\{ d \mid d^i = \sum_{s=1}^{S} z_s^i d_s^i, \sum_{s=1}^{S} z_s^i = 1, \sum_i \sum_{s=2}^{S} z_s^i \leq \Gamma, z_1^i \in \{0,1\} \quad \forall \ i, s \right\}
\]

where \( d \) is the demand vector realized for the network and \( d^i \) is the demand realized at demand point \( i \). Hereinafter, the superscript \( d \) denotes variables associated with a particular demand vector \( d \in D \).

4.2.3 List of Notations

Below is the list of some useful notations used for the proposed model in this chapter.

\( i \) index for demand points, \( i \in I \)

\( j \) index for shelter locations, \( j \in J \)
\(N\) set of nodes
\(A\) set of links
\(\Gamma\) degree of pessimism
\(B\) total number of available buses
\(b\) index for the buses
\(\beta_b\) capacity of bus \(b\)
\(T_{ij}\) round trip travel time from pick-up location \(i\) to shelter \(j\)
\(C_{ip}\) walking distance between the node pair \((i, p)\) in the network
\(\omega\) longest walking distance allowed from any demand point to its nearest pick-up location
\(T_{\text{max}}\) maximum running time allowed for each bus during evacuation
\(K_j\) capacity of shelter \(j\)
\(d\) demand vector
\(d^i\) demand realized for demand point \(i\)
\(d^i_s\) demand for demand point \(i\) under scenario \(s\)
\(q_p\) demand accumulated at the pick-up location \(p\)
\(z^i_s\) binary variable indicating which scenario \(s\) is realized for demand point \(i\)
\(\mu^i_b\) binary variable indicating bus allocation decision
\(\delta^i_p\) binary variable indicating evacuees choice of pick-up location
\(W_i\) distance of demand point \(i\) to its nearest pick-up locations
\(y_i\) binary variable indicating pick-up location decision
\(X_{bij}\) integer variable representing number of trips bus \(b\) makes between pick-up location \(i\) and shelter \(j\)
Note that, although not all the notations used in this chapter are listed here, they are defined appropriately wherever used.

4.3 Model Formulation

Let $G(N, A)$ denote the network, where $N$ and $A$ are the sets of nodes and links in the network respectively. Assume that the transit-dependent demand during evacuation is originated from a subset of the nodes, i.e., the set of demand points $I \subset N$. Next, let $J \subset N$ denote a set of shelter locations in the network. A binary variable $y_i$ represents the pick-up location decision, which is 1 if a pick-up point is located at $i \in I$ and 0 otherwise.

It is assumed that buses are the only public transportation available for evacuation. Let $B$ denote the number of buses available for the evacuation and $\beta_b$ denote the capacity of bus $b$. A binary variable $\mu_{bi}$ represents the bus allocation decision, which is 1 if bus $b$ is assigned to pick-up point $i$ and 0 otherwise. We use an integer variable $X_{bij}$ to represent the number of trips bus $b$ needs to make between pick-up point $i$ to shelter $j$. We further use a binary variable $\delta_{ip}$ to represent evacuees’ choice of pick-up points. The variable is 1 if pick-up point $p$ is nearest to the demand point $i$, and is thus selected. Otherwise, it is 0. Let $W_i$ denote the distance of demand point $i$ to its nearest pick-up location and $q_p$ the demand accumulated at the pick-up location $p$.

Using the above notations, a robust transit pick-up location and bus allocation model or RTPL can be formulated as follows:

RTPL
\[
\begin{align*}
\min & \quad \sum_{y,x,d,z,q,\delta,u} \sum_{b} \sum_{i} \sum_{j} T_{ij} X_{bij} \\
\text{s.t.} & \quad \sum_{b} \sum_{j} \beta_{b} X_{bij} \geq q^d_i \quad \forall \ i \in I, \ d \in D \\
& \quad \sum_{b} \sum_{i} \beta_{b} X_{bij} \leq K_j \quad \forall \ j \in J \\
& \quad \mu^i_b = 1 \quad \forall \ b \in \{1,2,\ldots,B\} \\
& \quad \mu^i_b \leq B y_i \quad \forall \ i \in I \\
& \quad \sum_{j} X_{bij} \leq M \mu^i_b \quad \forall \ i \in I, \ b \in \{1,2,\ldots,B\} \\
& \quad W_i = \sum_{p} \delta^i_p C_{ip} \quad \forall \ i, p \in I \\
& \quad W_i \leq C_{ip} y_p + M (1 - y_p) \quad \forall \ i, p \in I \\
& \quad \delta^i_p = 1 \quad \forall \ i, p \in I \\
& \quad \delta^i_p \leq y_p \quad \forall \ i, p \in I \\
& \quad q^d_p = \sum_{i} \delta^i_p d^i \quad \forall \ i, p \in I, \ d \in D \\
& \quad W_i \leq \omega \quad \forall \ i \in I \\
& \quad \sum_{j} T_{ij} X_{bij} \leq T_{\text{max}} \quad \forall \ b \in \{1,2,\ldots,B\} \\
& \quad y_i, \delta^i_p, \mu^i_b \in \{0,1\} \quad \forall \ i, p, b \\
X_{bij} \text{integer}
\end{align*}
\]

where $T_{ij}$ is the round-trip travel time from a pick-up point $i$ to a shelter $j$; $K_j$ is the capacity of public shelter $j$ available for transit-dependent evacuees; $C_{ip}$ is the distance between the node pair $(i,p)$ in the network and is an input to the model; $\omega$ is the longest walking distance allowed from any demand point to its nearest pick-up point; $T_{\text{max}}$ is the
maximum running time allowed for each bus during evacuation; and $M$ is a sufficiently large number.

The objective of RTPL is to minimize the total evacuation time. Constraint (4.1) ensures all the evacuees under all demand scenarios realized from the uncertainty set can be transported by buses. Constraint (4.2) requires the total number of evacuees transported to a shelter to be less than the available capacity at that shelter. Constraint (4.3) ensures that each bus is assigned to only one particular pick-up point. Constraint (4.4) implies that the node a bus is assigned to is a pick-up point. Constraint (4.5) specifies that a bus can make trips to different shelters but it only goes to its assigned pick-up point. Constraints (4.6)-(4.7) calculate the walking distance of a demand point to its nearest pick-up point. Constraint (4.8) assumes that all the evacuees from a particular demand point go to the same nearest pick-up point. Constraint (4.9) ensures that the node where evacuees assemble is a pick-up point. Constraint (4.10) is to estimate the total number of evacuees assembling at each pick-up point. Constraint (4.11) requires that the walking distance from each demand point to its nearest pick-up point should be less than the maximum allowed. Finally, constraint (4.12) imposes an upper bound on the total running time of each bus during evacuation.

Note that constraints (4.6)-(4.9) and (4.11) can be further simplified as the following equivalent set of constraints. We present as above to facilitate the understanding of the model structure.

$$W_i = \sum_p \delta_p^i c_{ip} \quad \forall \ i \in I, \ p \in I^+$$

$$W_i \leq c_{ip} y_p + M(1 - y_p) \quad \forall \ i \in I, \ p \in I^+$$

$$\sum_p \delta_p^i = 1 \quad \forall \ i \in I, \ p \in I^+$$
where $l^+ = \{i \mid C(i, k) \leq \omega \ \forall i, k \in I\}$

In summary, the output of the above model includes the optimal locations of pick-up points, i.e., $y$, allocation of buses to pick-up points, i.e., $\mu$, the number of trips made by each bus, i.e., $X$, evacuees' choice of pick-up points, i.e., $\delta$, and the total demand accumulated at each pick-up point, i.e., $q$.

### 4.4 Solution Algorithm

As formulated, RTPL is a mixed integer program. The number of demand vectors in the uncertainty set $D$ can be extremely large, which renders RTPL difficult to solve. A cutting plane scheme (Lawphongpanich and Hearn, 2004) is developed to solve RTPL. More specifically, a sequence of restricted RTPL problems are solved with respect to a subset of demand vectors $\overline{D} \subset D$ that contains a much smaller number of demand vectors and then another problem is solved to generate demand vectors one at a time as needed to expand $\overline{D}$.

The restricted RTPL problem (R-RTPL) can be written as follows:

$$\min_{y, \alpha, \mu} \sum_b \sum_{i} \sum_{j} T_{ij}X_{bij}$$

s.t.

$$\sum_{b} \sum_{j} \beta_{b}X_{bij} \geq q_i^d \quad \forall \ i \in I, d \in \overline{D} = \{d_1, d_2, ... d_n\}$$ (4.13)

$$q_i^d = \sum_{i} \delta_{p}^d \mu_{bi} \quad \forall \ i, p \in I, d \in \overline{D} = \{d_1, d_2, ... d_n\}$$ (4.14)

$$y_i, \delta_{p}^i, \mu_{bi} \in \{0,1\} \quad \forall \ i, p, b$$

$$X_{bij} \text{ integer}$$

(4.2)-(4.9), (4.11) and (4.12)
Compared with RTPL, the above formulation contains a less number of constraints, and is thus much easier to solve. Let \((\hat{y}, \hat{\delta}, \hat{x})\) solve the R-RTPL globally with the optimal objective value as \(\hat{\xi}\). Then the solution \((\hat{y}, \hat{\delta}, \hat{x})\) also solves the original RTPL problem if the bus allocation plan \(\hat{x}\) is sufficient to transport the demands at pick-up points \(\hat{y}\) under all demand scenarios in \(D\). Mathematically, this can be verified by solving the following worst-case demand problem (WCD):

\[
\text{WCD} \\
\max_{z,d} \sum_i e_i \\
\text{s.t.} \\
e_i \leq \eta_i M \quad \forall \ i \in I \\
e_i - (q_i - \sum_b \sum_f \beta_{bf} \hat{x}_{bf}) \leq (1 - \eta_i) M \quad \forall \ i, s \\
e_i \geq 0 \quad \forall \ i \\
q_p = \sum_i \hat{\delta}_p^i \hat{d}_i^i \quad \forall \ i, p \in I \\
d_i^i = \sum_{s=1}^{S_i} z_s^i d_s^i \quad \forall \ i, s \\
\sum_{s=1}^{S_i} z_s^i = 1, \quad \forall \ i, s \\
\sum_i \sum_{s=2}^{S_i} z_s^i \leq \Gamma \quad \forall \ i, s \\
z_s^i, \eta_i \in \{0,1\} \quad \forall \ i, s
\]

where \(e_i\) denote the excess demand at pick-up point \(i\) and \(M\) is a sufficiently large number.

The objective of WCD is to find a demand vector \(d\) that maximizes the total excess demand, i.e., the number of evacuees not transported by the current bus allocation plan. Constraints (4.15)-(4.18) are to calculate the excess demand at pick-up point \(i\).
Constraint (4.19) calculates the total number of evacuees at each pick-up point.

Constraints (4.20)-(4.21) ensure that the demand vector \( d \) belongs to the demand uncertainty set.

If the optimal objective value of WCD is no greater than zero, then the current bus allocation plan is sufficient, i.e., \( (\hat{y}, \hat{\delta}, \hat{X}) \) is the optimal solution to the original RTPL problem. Otherwise, an improved solution may be obtained by expanding the demand vector set \( \bar{D} \cup \{\hat{d}\} \), where \( \hat{d} \) is the solution to the above WCD. Below is the procedure of the cutting plane scheme outlined above.

Cutting Plane Algorithm:

Step 0: Choose an initial demand vector \( d_1 \in D \). Set \( n = 1 \) and \( \bar{D} = \{d_1\} \).

Step 1: Solve the R-RTPL problem with a subset of demand vectors \( \bar{D} \) and let \( (\hat{y}^n, \hat{\delta}^n, \hat{X}^n) \) denote the resulting optimal solution.

Step 2: Solve the WCD problem associated with the current location of pick-up points \( \hat{y}^n \) and the bus allocation \( \hat{X}^n \). Let \( d_n \) denote the optimal solution and \( \hat{W} \) denote the corresponding objective value.

Step 3: If \( \hat{W} \leq \varepsilon \), stop and \( (\hat{y}^n, \hat{\delta}^n, \hat{X}^n) \) is an optimal solution to the RTPL problem. Otherwise, set \( \bar{D} = \bar{D} \cup \{d_n\}, n = n + 1 \) and go to step 1.

Because the number of demand vectors in the uncertainty set \( D \) is finite, the cutting plane algorithm terminates after a finite number of iterations.

4.5 Numerical Example

This section presents results from solving the proposed model on the Sioux Falls network (LeBlanc et al., 1975). The cutting plane algorithm is implemented in GAMS.
(Brooke et al., 1992) where the R-RTPL and WCD problems are solved using CPLEX (CPLEX, 2008).

For the Sioux Falls network, the original data for each link consist of two parameters and they are transformed into the link’s free-flow travel time and capacity. Mass evacuation is considered for a hypothetical disaster in the region. As shown in Figure 4-1, we assume that nodes 1-12 and 16-18 are the demand points and public shelters are located at nodes 13, 20, 21 and 22. It is further assumed that the planning agency has estimated three possible demand values at each demand point, i.e., nominal, low and high. The nominal demand is reported in Table 4-1 and the other two demand values, low and high, are generated by multiplying the nominal demand by a uniformly-distributed number between (0.5, 1) and (1.5, 2). The shelter location and capacity are reported in Table 4-2. It is further assumed that there are 10 buses available for evacuation and each has a capacity of 30. The maximum allowable running time of each bus is 180 min and the maximum allowable walking time from a demand point to a pick-up point is 5 min.
Figure 4-1. Sioux Falls network with demand points and known shelter locations
Table 4-1. Demand values at each demand point

<table>
<thead>
<tr>
<th>Demand Points</th>
<th>Nominal Demand</th>
<th>Low Demand</th>
<th>High Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 1</td>
<td>60.00</td>
<td>35.15</td>
<td>109.20</td>
</tr>
<tr>
<td>Node 2</td>
<td>42.00</td>
<td>38.70</td>
<td>66.35</td>
</tr>
<tr>
<td>Node 3</td>
<td>40.00</td>
<td>31.00</td>
<td>65.00</td>
</tr>
<tr>
<td>Node 4</td>
<td>46.00</td>
<td>29.92</td>
<td>84.38</td>
</tr>
<tr>
<td>Node 5</td>
<td>38.00</td>
<td>24.55</td>
<td>65.28</td>
</tr>
<tr>
<td>Node 6</td>
<td>50.00</td>
<td>30.60</td>
<td>84.00</td>
</tr>
<tr>
<td>Node 7</td>
<td>34.00</td>
<td>22.95</td>
<td>56.98</td>
</tr>
<tr>
<td>Node 8</td>
<td>44.00</td>
<td>40.84</td>
<td>68.90</td>
</tr>
<tr>
<td>Node 9</td>
<td>44.00</td>
<td>23.48</td>
<td>69.30</td>
</tr>
<tr>
<td>Node 10</td>
<td>52.00</td>
<td>39.00</td>
<td>93.32</td>
</tr>
<tr>
<td>Node 11</td>
<td>48.00</td>
<td>47.96</td>
<td>91.95</td>
</tr>
<tr>
<td>Node 12</td>
<td>40.00</td>
<td>31.58</td>
<td>64.62</td>
</tr>
<tr>
<td>Node 16</td>
<td>36.00</td>
<td>35.84</td>
<td>65.98</td>
</tr>
<tr>
<td>Node 17</td>
<td>26.00</td>
<td>22.90</td>
<td>49.10</td>
</tr>
<tr>
<td>Node 18</td>
<td>30.00</td>
<td>16.96</td>
<td>49.55</td>
</tr>
<tr>
<td>Total Demand</td>
<td>630.00</td>
<td>471.43</td>
<td>1083.91</td>
</tr>
</tbody>
</table>

Table 4-2. Available capacity of public shelters

<table>
<thead>
<tr>
<th>Public Shelters</th>
<th>Available Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 13</td>
<td>240.00</td>
</tr>
<tr>
<td>Node 20</td>
<td>333.00</td>
</tr>
<tr>
<td>Node 21</td>
<td>360.00</td>
</tr>
<tr>
<td>Node 22</td>
<td>300.00</td>
</tr>
<tr>
<td>Total Capacity</td>
<td>1230.00</td>
</tr>
</tbody>
</table>

Tables 4-3 and 4-4 present the optimal locations of pick-up points and allocation of bus trips for the nominal, robust and worst-case plans. The robust plan is obtained using the degree of pessimism, $\Gamma = 3$ while the nominal plan is obtained with $\Gamma = 0$ (deterministic demand with nominal value). The worst-case plan is the optimal plan against deterministic high demand at each demand point. It is equivalent, in this particular example, to solving the proposed model with the degree of pessimism $\Gamma = 15$. 
In order to evaluate the performance of these three different plans, 1000 demand vectors are randomly generated by assuming that, for each demand point, there is an equal probability of occurrence for the nominal, low and high demand values. For each sampled demand vector, it is then evaluated that whether the bus allocation is sufficient to transport all the evacuees at pick-up points. The probability of meeting the demand is calculated accordingly and reported in Table 4-3.

Table 4-3. Pick-up points under nominal, robust and worst-case plans

<table>
<thead>
<tr>
<th>Pick-up Locations</th>
<th>Nominal Plan</th>
<th>Robust Plan($\Gamma = 3$)</th>
<th>Worst-Case Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Node 3</td>
<td>Node 3</td>
<td>Node 3</td>
</tr>
<tr>
<td></td>
<td>Node 6</td>
<td>Node 6</td>
<td>Node 6</td>
</tr>
<tr>
<td></td>
<td>Node 10</td>
<td>Node 10</td>
<td>Node 10</td>
</tr>
<tr>
<td></td>
<td>Node 18</td>
<td>Node 17</td>
<td>Node 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Node 18</td>
<td>Node 17</td>
</tr>
</tbody>
</table>

| Total Evacuation Time (min) | 612.00 | 1128.80 | 1234.20 |
| Probability of Meeting Demand with Allocated Bus Trips | 2.13 % | 97.94 % | 100.00 % |
Table 4-4. Bus allocation under nominal, robust and worst-case plans

<table>
<thead>
<tr>
<th>Bus #</th>
<th>Trips</th>
<th>Pick-up Location</th>
<th>Shelter</th>
<th>Bus #</th>
<th>Trips</th>
<th>Pick-up Location</th>
<th>Shelter</th>
<th>Bus #</th>
<th>Trips</th>
<th>Pick-up Location</th>
<th>Shelter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>Node 18</td>
<td>Node 20</td>
<td>1</td>
<td>2</td>
<td>Node 6</td>
<td>Node 20</td>
<td>1</td>
<td>5</td>
<td>Node 10</td>
<td>Node 22</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>Node 3</td>
<td>Node 13</td>
<td>2</td>
<td>4</td>
<td>Node 10</td>
<td>Node 22</td>
<td>2</td>
<td>3</td>
<td>Node 6</td>
<td>Node 21</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>Node 10</td>
<td>Node 22</td>
<td>2</td>
<td>1</td>
<td>Node 10</td>
<td>Node 21</td>
<td>3</td>
<td>4</td>
<td>Node 18</td>
<td>Node 20</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Node 6</td>
<td>Node 20</td>
<td>3</td>
<td>4</td>
<td>Node 10</td>
<td>Node 22</td>
<td>4</td>
<td>3</td>
<td>Node 6</td>
<td>Node 20</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>Node 6</td>
<td>Node 20</td>
<td>4</td>
<td>7</td>
<td>Node 3</td>
<td>Node 13</td>
<td>5</td>
<td>1</td>
<td>Node 3</td>
<td>Node 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>4</td>
<td>Node 6</td>
<td>Node 20</td>
<td>5</td>
<td>1</td>
<td>Node 3</td>
<td>Node 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>1</td>
<td>Node 17</td>
<td>Node 20</td>
<td>6</td>
<td>4</td>
<td>Node 6</td>
<td>Node 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>2</td>
<td>Node 17</td>
<td>Node 22</td>
<td>7</td>
<td>7</td>
<td>Node 3</td>
<td>Node 13</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>4</td>
<td>Node 18</td>
<td>Node 20</td>
<td>8</td>
<td>4</td>
<td>Node 17</td>
<td>Node 22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>3</td>
<td>Node 6</td>
<td>Node 21</td>
<td>9</td>
<td>3</td>
<td>Node 12</td>
<td>Node 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>1</td>
<td>Node 3</td>
<td>Node 13</td>
<td>10</td>
<td>3</td>
<td>Node 10</td>
<td>Node 21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>2</td>
<td>Node 3</td>
<td>Node 21</td>
<td>10</td>
<td>1</td>
<td>Node 10</td>
<td>Node 22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>1</td>
<td>Node 17</td>
<td>Node 21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 4-3, the difference in total evacuation time and the reliability of the nominal, robust and worst-case plans can be observed. The worst-case plan is 100% reliable in meeting the demand realized under uncertainty but the estimated total evacuation time (and the required bus running time, and the associated resource) is very high. On the other hand, the robust plan requires less total evacuation time, saving approximately 9% of the evacuation time (and the associated resource) while achieving nearly the same level of reliability in meeting the realized demand. The nominal plan yields a lot of saving in total evacuation time, but meets the evacuation demand with a probability of 2%. Such an exceptionally low reliability underscores the importance of accommodating the demand uncertainty in evacuation planning.

Table 4-5 compares the nominal and worst-case plans against the robust plans with varying the degree of pessimism. In general, the probability of a robust plan in meeting the demand increases with the increase in the degree of pessimism $\Gamma$, but the total evacuation time estimate also increases. As previously mentioned, $\Gamma$ controls the
degree of conservatism in the solution. An appropriate value of degree of pessimism can be chosen depending on the decision makers’ attitude towards risk.

Table 4-5. Reliability of plans with varying degree of pessimism

<table>
<thead>
<tr>
<th>Degree of Pessimism (I')</th>
<th>Total Evacuation Time (min)</th>
<th>Probability of Meeting Travel Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (nominal case)</td>
<td>612.00</td>
<td>2.13 %</td>
</tr>
<tr>
<td>1</td>
<td>782.00</td>
<td>34.88 %</td>
</tr>
<tr>
<td>2</td>
<td>965.60</td>
<td>76.42 %</td>
</tr>
<tr>
<td>3</td>
<td>1128.80</td>
<td>97.94 %</td>
</tr>
<tr>
<td>4</td>
<td>1181.00</td>
<td>99.59 %</td>
</tr>
<tr>
<td>5</td>
<td>1227.40</td>
<td>100.00 %</td>
</tr>
<tr>
<td>6</td>
<td>1227.40</td>
<td>100.00 %</td>
</tr>
<tr>
<td>7</td>
<td>1227.40</td>
<td>100.00 %</td>
</tr>
<tr>
<td>8</td>
<td>1234.20</td>
<td>100.00 %</td>
</tr>
<tr>
<td>15 (worst-case)</td>
<td>1234.20</td>
<td>100.00 %</td>
</tr>
</tbody>
</table>

4.6 Summary

This chapter proposed a robust optimization approach to determine the optimal locations of transit pick-up points and trip allocations for buses in transit-based evacuation planning under demand uncertainty. The proposed model was formulated as a mixed integer program and solved by a cutting plane scheme. Numerical results from the Sioux Falls network illustrate that the proposed robust model produced a less conservative but reliable plan, compared with the one optimized against the worst-case demand. In addition, importance of incorporating the demand uncertainty in evacuation planning was also demonstrated.
Although only demand uncertainty is considered, the proposed model framework is general enough to accommodate other types of uncertainty. For future studies, we plan to implement the proposed model and solution algorithm for a large-scale realistic network. In addition, model can be further extended to capture the time-dependent nature of evacuation into the framework.
CHAPTER 5  
INTEGRATED APPROACH FOR PUBLIC SHELTERS AND TRANSIT PICK-UP LOCATIONS WITH DEMAND UNCERTAINTY

The previous chapter discussed a modeling approach to determine the optimal pick-up locations for transit evacuation, where the locations of public shelters are assumed to be known and given. However, along with pick-up locations, it is critical that the shelter location decision be made carefully based on the different residential locations of transit-dependent evacuees. For instance, studies have shown that households with higher income tend to evacuate to motels/hotels and households with higher education and/or, with pets and children are more likely to stay with families and friends (Whitehead et al., 2000). It is found that the low-income and minority families or elderly people are the ones who are more likely to evacuate to shelters (Mileti et al., 1992; Whitehead et al., 2000; Willigen et al., 2002). In general, it can be said that significant portion of evacuee population that are more likely to stay at public shelters are transit-dependent residents. Therefore it would be more realistic to determine the optimal shelter locations based on the residential locations of transit-dependent evacuees, which in turn could potentially affect the optimal locations of transit pick-up points. In other words, optimal locations of shelters, residential locations of transit-dependent evacuees, and optimal transit pick-up locations could be mutually interconnected. Consequently, it is imperative to consider an integrated approach to model shelter locations and pick-up locations simultaneously for transit evacuation. Thus, in this chapter, the modeling approach for transit evacuation planning is further enhanced and extended it in two aspects: we determine both pick-up and shelter locations under demand uncertainty and consider a two-stage robust optimization approach to achieve flexible and less conservative results.
For the remainder of this chapter, Section 5.1 describes the model considerations, which are fairly similar to those discussed in Chapter 4. Section 5.2 presents the model formulation based on single-stage robust optimization approach. Section 5.3 then presents the two-stage robust optimization approach. An efficient solution algorithm to solve two-stage problem is proposed in Section 5.4. A numerical example is given in Section 5.5 to illustrate the effectiveness of proposed model and compare the results from the two approaches. Summary of this chapter is provided in Section 5.6.

5.1 Model Considerations

5.1.1 Basic Assumptions

For transit evacuation, it is assumed that the transit-dependent evacuees will assemble at designated pick-up locations and the transit vehicles will pick up and transport them to the public shelters. But, now the location of public shelters is also considered a decision variable, unlike the pick-up location model discussed in Chapter 4. Thus our aim is to determine the optimal shelter locations, pick-up locations, allocation of given transit vehicles and the capacities of public shelters.

In our modeling framework, pick-up locations are modeled in a way similar to the pick-up location problem of Chapter 4. It is assumed that the evacuees from different demand points go to the nearest pick-up locations and assemble there before the start of evacuation process. For modeling the public shelters, we consider that the potential sites where shelters can be located are given and known, and there are a maximum number of shelters which can be opened out of all the potential sites based on severity of the disaster.

The number of available transit vehicles and their capacities are assumed to be known and each transit vehicle is assigned to a particular pick-up location and the
vehicle can make trips to different shelters but serves only its assigned pick-up location. It is further assumed that transit vehicles, in a limited number, have negligible impact on overall congestion, and thus the travel time between pick-up points and public shelters are given and constant.

Numbers of transit-dependent evacuees at different residential locations are considered to be uncertain. A similar approach as discussed earlier is used to model demand uncertainty and the same uncertainty set is used to characterize the transit-dependent evacuation demand, i.e.,

\[ D = \{ d^i | d^i = \sum_{s=1}^{S^i} z^i_s d^i_s, \sum_{s=1}^{S^i} z^i_s = 1, \sum_{i=1}^{S^i} z^i_s \leq \Gamma, z^i_s \in \{0,1\} \; \forall \; i, s \} \]

where \( d \) is the demand vector realized for the network and \( d^i \) is the demand realized at demand point \( i \).

Let \( G(N,A) \) denote the network, where \( N \) and \( A \) are the sets of nodes and links in the network respectively. Assume that the transit-dependent demand during evacuation is originated from a subset of the nodes, i.e., the set of demand points \( I \subseteq N \). Next, let \( J \subseteq N \) denote a set of potential sites to locate shelters in the network. A binary variable \( y_i \) represents the pick-up location decision, which is 1 if a pick-up point is located at \( i \in I \) and 0 otherwise. A binary variable \( \alpha_j \) represents the shelter location decision, which is 1 if a shelter is located at node \( j \in J \) and 0 otherwise.

Let \( B \) denote the number of buses available for the evacuation and \( \beta_b \) denote the capacity of bus \( b \). A binary variable \( \mu^i_b \) represents the bus allocation decision, which is 1 if bus \( b \) is assigned to pick-up point \( i \) and 0 otherwise. We use an integer variable \( X_{b_{ij}} \) to represent the number of trips bus \( b \) needs to make between pick-up location \( i \) to shelter \( j \). We further use a binary variable \( \delta^i_{p} \) to represent evacuees’ choice of pick-up.
points. The variable is 1 if pick-up point $p$ is nearest to the demand point $i$, and is thus selected; otherwise, it is 0. Let $W_i$ denote the distance of demand point $i$ to its nearest pick-up location. Furthermore, $q_p$ is the demand accumulated at the pick-up location $p$ and $K_j$ denote the capacity of shelter $j$.

5.1.2 List of Notations

Below is the list of some useful notations used for the proposed model in this chapter.

- $i$ index for demand points, $i \in I$
- $j$ index for shelter locations, $j \in J$
- $N$ set of nodes
- $A$ set of links
- $\Gamma$ degree of pessimism
- $B$ total number of available buses
- $b$ index for the buses
- $\beta_b$ capacity of bus $b$
- $T_{ij}$ round trip travel time from pick-up location $i$ to shelter $j$
- $C_{ip}$ walking distance between the node pair $(i, p)$ in the network
- $\omega$ longest walking distance allowed from any demand point to its nearest pick-up location
- $T_{\text{max}}$ maximum running time allowed for each bus during evacuation
- $d$ demand vector
- $d^i$ demand realized for demand point $i$
- $d^i_s$ demand for demand point $i$ under scenario $s$
\[ q_p \] demand accumulated at the pick-up location \( p \)

\[ z^i_s \] binary variable indicating which scenario \( s \) is realized for demand point \( i \)

\[ \mu^i_b \] binary variable indicating bus allocation decision

\[ \delta^i_p \] binary variable indicating evacuees choice of pick-up location

\[ W_i \] distance of demand point \( i \) to its nearest pick-up locations

\[ K_j \] capacity of shelter \( j \)

\[ \alpha_j \] binary variable indicating shelter location decision

\[ \gamma_i \] binary variable indicating pick-up location decision

\[ X_{bij} \] integer variable representing number of trips bus \( b \) makes between pick-up location \( i \) and shelter \( j \)

Note that, although not all the notations used in this chapter are listed here, they are defined appropriately wherever used.

### 5.2 Robust Optimization Approach

The robust pick-up and shelter locations (RPSL) can be obtained by solving the following mathematical program:

\[
\text{RPSL} \quad \min_{y, \alpha, X, K, d, q, \delta, \mu} \sum_b \sum_i \sum_j T_{ij} X_{bij}
\]

s.t.

\[ \sum_b \sum_j \beta^i_b X_{bij} \geq q^d_i \quad \forall \ i \in I, \ d \in D \quad (5.1) \]

\[ \sum_b \sum_i \beta^i_b X_{bij} \leq K_j \quad \forall \ j \in J \quad (5.2) \]

\[ \sum_i \mu^i_b = 1 \quad \forall \ b \in \{1, 2, ..., B\} \quad (5.3) \]

\[ \sum_b \mu^i_b \leq By_i \quad \forall \ i \in I \quad (5.4) \]
\[
\sum_j X_{bij} \leq L_1 \mu_b^i \quad \forall i \in I, b \in \{1,2, \ldots, B\}
\] (5.5)

\[
\sum_i X_{bij} \leq L_2 \alpha_j \quad \forall j \in J, b \in \{1,2, \ldots, B\}
\] (5.6)

\[
\sum_j \alpha_j \leq M
\] (5.7)

\[
W_i = \sum_p \delta_p^i C_{ip} \quad \forall i, p \in I
\] (5.8)

\[
W_i \leq C_{ip} y_p + L_3 (1 - y_p) \quad \forall i, p \in I
\] (5.9)

\[
\sum_p \delta_p^i = 1 \quad \forall i, p \in I
\] (5.10)

\[
\delta_p^i \leq y_p \quad \forall i, p \in I
\] (5.11)

\[
W_i \leq \omega \quad \forall i \in I
\] (5.12)

\[
q_p^d = \sum_i \delta_p^i d_i^d \quad \forall i, p \in I, d \in D
\] (5.13)

\[
\sum_i \sum_j T_{ij} X_{bij} \leq R_{max} \quad \forall b \in \{1,2, \ldots, B\}
\] (5.14)

\[
\alpha_j, y_i, \delta_p^i, \mu_b^i \in \{0,1\} \quad \forall i, j, p, b
\]

\(X_{bij}\) integer

where \(T_{ij}\) is the round-trip travel time from a pick-up point \(i\) to a shelter \(j\); \(C_{ip}\) is the distance between the node pair \((i, p)\) in the network and is an input to the model; \(\omega\) is the longest walking distance allowed from any demand point to its nearest pick-up point; \(R_{max}\) is the maximum running time allowed for each bus during evacuation; \(M\) is the maximum number of shelter which can be opened; and \(L_1, L_2, L_3\) are sufficiently large numbers.

In the above RPSL, the objective function is to minimize the total evacuation time. Constraint (5.1) ensures that all the evacuees under each demand scenario realized from the uncertainty set can be transported by the allocated buses. Constraint (5.2) requires the available capacity of public shelter to be greater than the total number of
evacuees transported to that particular shelter. Constraint (5.3) ensures that each bus is assigned to only one particular pick-up point. Constraint (5.4) implies that the node a bus is assigned to is a pick-up point. Constraint (5.5) specifies that a bus can make trips to different shelters but it only goes to its assigned pick-up point. Constraint (5.6) specifies that a bus make trips to only open shelters. Constraint (5.7) imposes a bound on the maximum number of shelters which can be opened. Constraints (5.8)-(5.9) calculate the walking distance of a demand point to its nearest pick-up location. Constraint (5.10) assumes that all the evacuees from a particular demand point go to the same nearest pick-up location. Constraint (5.11) ensures that the node where evacuees assemble is a pick-up location. Constraint (5.12) requires that the walking distance from each demand point to its nearest pick-up location should be less than the maximum required. Constraint (5.13) estimates the total number of evacuees assembling at each pick-up location. Finally, constraint (5.14) imposes an upper bound on the total running of each bus during evacuation.

The above model is a straightforward extension of the pick-up location model proposed in Chapter 4. The model output includes the optimal locations of public shelters, i.e., $\alpha$, capacities of public shelters, i.e., $K$, optimal locations of pick-up points, i.e., $y$, allocation of buses to pick-up points, i.e., $\mu$, the number of trips made by each bus, i.e., $X$, evacuees’ choice of pick-up points, i.e., $\delta$, and the total demand accumulated at each pick-up point, i.e., $q$. The RPSL model can be solved using a similar cutting plane scheme as proposed in Section 4.4.

One of the modeling limitations of the above single-stage robust optimization is to make all the decisions before the realization of uncertain parameters, and thus the
solution tends to be conservative. This is a disadvantage of single-stage robust optimization in the situations when the decisions can be made sequentially and some of the decision variables can adjust themselves, to some extent, to the true value of the uncertain data. The proposed single stage robust optimization model (RSPL) does not consider the fact that in transit evacuation situation, decisions are made at different stages and consequently produces too conservative estimates. In order to address this issue and obtain a less conservative solution, a two-stage robust optimization approach is proposed next for optimal shelter and pick-up locations for transit evacuation planning.

5.3 Two-Stage Robust Optimization Approach

Robust optimization is a recent approach to optimization under uncertainty. To hedge against uncertainty, a robust optimization approach constructs a single solution that is feasible for all the possible realization of the uncertain data. However, it is possible that because a robust (single-stage) optimization approach tries to optimize over the worst-case scenario, it may produce conservative solutions. In addition, in single-stage robust optimization approaches, decision-maker has to obtain a solution before the real values of uncertain parameters are observed and therefore do not allow for recourse actions that may be based on realized values of some of these parameters. This disadvantage prevents its application to the problems where decisions are made sequentially.

To address this drawback of single-stage robust optimization, two-stage robust optimization (more general, multi-stage) also known as robust adjustable or adaptable optimization, has been introduced and studied (e.g., Ben-Tal et al., 2004). Two-stage robust optimization, an extension of robust optimization, aims to solve mathematical
programming problems where the data is uncertain and decisions can be made at
different points in time, thus producing solutions that are less conservative in nature
than produced by one-stage robust optimization. It is possible that in some cases, one-
stage and two-stage robust optimization approaches are equivalent. However, the
reason to explore two-stage approach is to enable more flexibility in cases when the
one-stage approach is unjustifiably conservative.

The basic idea behind the two-stage robust optimization approach is to consider
two set of variables, such that the first set of variables are determined before the
realization of uncertainty, and the second set of variables are derived after the
uncertainty is revealed. Such a framework enables the decision makers to fully take
advantage of the revealed information to make less conservative decisions. Two-stage
robust optimization is becoming more popular for decision making under uncertainty
and some of the recent applications include network/transportation problems (e.g.,
Atamturk and Zhang, 2007; Ordonez and Zhao, 2007; Gabrel et al., 2011; Zeng, 2011),
portfolio optimization (Takeda, 2008) and power system control and scheduling (Zhao
and Zeng, 2010; Bertsimas et al., 2011; Jiang et al., 2011).

5.3.1 General Modeling Approach for Two-Stage Problem

In this sub-section, a general modeling approach for two-stage robust optimization
problem (e.g., Zeng, 2011) is provided to facilitate the understanding of our proposed
model. Let us consider a linear optimization problem formulated as:

\[
\begin{align*}
\min & \quad cy + dx \\
\text{s.t.} & \quad Ay + Bx = u \\
& \quad y \in S_y, x \geq 0
\end{align*}
\]
where, $y$ is the first-stage and $x$ is the second-stage variables respectively, $c$ is the first-stage cost, $d$ is the second-stage cost, $u$ is the uncertain parameter belonging to uncertainty set $U$, $S_y$ is a polyhedron described by a finite list of inequalities.

In a two-stage problem, the objective is to minimize the total cost, which includes first-stage cost and worst-case second stage cost over the uncertainty set. The general form of the two-stage robust optimization formulation can thus be written as:

$$\min_y cy + \max_{u \in U} \min_{x \in F(y,u)} dx$$

s.t. $y \in S_y$ and $F(y,u) = \{x \in S_x : Bx = u - Ay\}$

Once the first-stage decisions are obtained, and the realization of the uncertainty is known, the second stage problem can be defined as:

$$Q(y,u) = \min dx$$

s.t. $Bx = u - Ay$, $x \geq 0$

Therefore, the two-stage problem can also be written as follows:

$$\min_y cy + \max_{u \in U} Q(y,u)$$

s.t. $y \in S_y$

### 5.3.2 Two-Stage Robust Formulation

We now extend our single-stage robust optimization formulation, i.e., RPSL, and propose a two-stage robust optimization formulation. In the context of pick-up and shelter location problem and with the uncertainty set for the demand, note that the whole decision process can be decomposed into two steps. In the first stage, optimal locations of pick-up points and public shelters are determined and once the realized demands are known, bus-allocation and shelter capacities are decided in the second stage.
The model considerations are same as discussed above. The only difference in
the modeling framework is that rather than having one set of variables, now there are
second-stage variables corresponding to each demand vector in the uncertainty set.
Thus, the set of constraints which involves second-stage variables are written for each
demand vector in the set. Optimal locations of public shelters, i.e., $\alpha$, optimal locations of
pick-up points, i.e., $\gamma$, and evacuees’ choice of pick-up points, i.e., $\delta$ are the first-stage
decision variables. Allocation of buses to pick-up points, i.e., $\mu$, the number of trips
made by each bus, i.e., $\lambda$, capacity of public shelters, i.e., $K$, and the total demand
accumulated at each pick-up point, i.e., $q$ are the second-stage variables in the model.
The superscript $d$ denotes variables associated with a particular demand vector $d \in D$.

The two-stage robust pick-up and shelter location problem or TS-RPSL can be
formulated as the following mathematical program:

$$\min_{y, \alpha, \delta} \left[ \max_{d \in D} \left( \min_{\lambda, q, \mu} \sum_{b} \sum_{i} \sum_{j} T_{ij} x_{bij}^{d} \right) \right]$$

s.t.

$$\sum_{j} \sum_{i} \beta_{b} x_{bij}^{d} \geq q_{i}^{d} \quad \forall i \in I, d \in D \quad (5.15)$$

$$\sum_{i} \sum_{j} \beta_{b} x_{bij}^{d} \leq K_{j}^{d} \quad \forall j \in J, d \in D \quad (5.16)$$

$$\lambda_{b}^{d} = 1 \quad \forall b \in \{1, 2, \ldots, B\}, d \in D \quad (5.17)$$

$$\sum_{b} \lambda_{b}^{d} \leq B y_{i} \quad \forall i \in I, d \in D \quad (5.18)$$

$$\sum_{i} x_{bij}^{d} \leq L_{1} \lambda_{b}^{d} \quad \forall i \in I, b \in \{1, 2, \ldots, B\}, d \in D \quad (5.19)$$

$$\sum_{i} x_{bij}^{d} \leq L_{2} \alpha_{j} \quad \forall j \in J, b \in \{1, 2, \ldots, B\}, d \in D \quad (5.20)$$

$$\sum_{j} \alpha_{j} \leq M \quad (5.21)$$
\[ W_i = \sum_p \delta_p^i C_{ip} \quad \forall \, i, p \in I \]  
(5.22)

\[ W_i \leq C_{ip} y_p + L_3(1 - y_p) \quad \forall \, i, p \in I \]  
(5.23)

\[ \sum_p \delta_p^i = 1 \quad \forall \, i, p \in I \]  
(5.24)

\[ \delta_p^i \leq y_p \quad \forall \, i, p \in I \]  
(5.25)

\[ W_i \leq \omega \quad \forall \, i \in I \]  
(5.26)

\[ q_p^d = \sum_i \delta_p^i d^i \quad \forall \, i, p \in I, d \in D \]  
(5.27)

\[ \sum_i \sum_j T_{ij} x_{bij}^d \leq R_{max} \quad \forall \, b \in \{1, 2, ..., B\}, d \in D \]  
(5.28)

\[ x_{bij}^d \text{ integer} \]

The objective function for the above TS-RPSL model is to minimize the worst-case minimum evacuation time realized over the uncertainty set. Constraints are defined similar to the corresponding set of constraints in the RPSL model. Constraints (5.15)-(5.20) and (5.27)-(5.28) involves the second-stage variables, which are now corresponding to each demand vector in the uncertainty set. The set of constraints (5.21)-(5.26), which involves only the first-stage variables, remains unchanged.

### 5.4 A Cutting-Plane Algorithm for the Two-Stage Problem

#### 5.4.1 Solving Two-Stage Robust Model

Although two-stage robust optimization approach enables more flexibility compared to the single-stage approach, however, there is a price to pay for this flexibility, i.e., it results in additional complexity. In general, two-stage robust optimization leads to computationally intractable problems and are very difficult to solve. As shown in Ben-Tal et al. (2004), even a simple two-stage robust optimization problem could be NP-hard and become intractable. In the literature, two types of solution
approaches have been proposed to solve two-stage robust optimization problem. The first one is approximation algorithm, which assumes the second stage variables to be simple functions (e.g., affine functions) of the uncertainty (e.g., Bertsimas et al., 2008). However, when the structure of second-stage decision problem is not simple, such an approximation approach may not be applicable. The second type of algorithms are various cutting plane based methods, e.g., Benders-dual cutting plane algorithms (Gabrel et al., 2011; Jiang et al., 2011), column and constraint generation algorithm (Zhao and Zeng, 2010; Zeng, 2011), and Kelley’s algorithm (Thiele et al., 2009), which seek to derive the exact solutions.

In this section, a cutting plane scheme to solve two-stage robust problem (TS-RPSL) is presented. The algorithm is implemented in a master-subproblem framework, in which a sequence of master problems are solved with respect to a subset of demand vectors $\bar{D} \subset D$ and then another subproblem is solved to generate demand vectors one at a time as needed to expand $\bar{D}$.

The master problem is to find the optimal first-stage variables, i.e., pick-up locations and shelter locations, for a set of known demand vectors. The master problem (MP) is written as follows:

$$\min_{y, \alpha, \delta} \eta$$

s.t.

$$\eta \geq \sum_b \sum_i \sum_j T_{ij} X^d_{bij} \quad \forall d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (5.29)$$

$$\sum_b \sum_j \beta_b X^d_{bij} \geq q^d_i \quad \forall i \in I, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (5.30)$$

$$\sum_b \sum_i \beta_b X^d_{bij} \leq K^d_j \quad \forall j \in J, d \in \bar{D} = \{d_1, d_2, \ldots, d_n\} \quad (5.31)$$
The objective of above MP model is to find a pickup and shelter location plan, which minimizes \( \eta \), where \( \eta \) is the maximum of minimum evacuation times corresponding to a set of known demand vectors in \( \bar{D} \). Constraint (5.29) ensures that \( \eta \) is the maximum of evacuation times for different demand vectors. The expression on the right-hand side of constraint (5.29) is the total evacuation time value. Since the objective is to minimize the value of \( \eta \), the solution will force the total evacuation time to be minimum. More specifically, constraint (5.29) ensures the total evacuation time value is the minimum corresponding to the demand vector for which the constraint is binding, but not necessarily the minimum evacuation time for the rest demand vectors in the set. Since we only require minimizing the maximum of minimum evacuation time, this formulation will give us the desired location plan. As formulated, the above master problem is a mixed integer program (MIP) and can be solved directly using CPLEX (CPLEX, 2008).
Let \( (\hat{y}, \hat{\alpha}, \hat{\delta}) \) be the optimal solution to the above master problem with the objective function value being \( \xi \). The objective function value \( \xi \) is the minimum evacuation time for the given set of demand vectors. The obtained solution will be an optimal solution to the original TS-RPSL, if for the pick-up \( (\hat{y}) \) and shelter location \( (\hat{\alpha}) \) plan, there does not exist any other demand vector in \( D \) for which the minimum evacuation time is higher than \( \xi \). Mathematically, this can be verified by solving the following subproblem (SP).

\[
Q(\hat{y}, \hat{\alpha}, \hat{\delta}) = \max_{d,z} \min_{x,K,q} \sum_{b} \sum_{i} \sum_{j} T_{ij} X_{b ij}
\]

s.t.

\[
\sum_{b} \sum_{j} \beta_{b} X_{b ij} \geq q_{i} \quad \forall \; i \in I
\]

\[
\sum_{b} \sum_{i} \beta_{b} X_{b ij} \leq K_{j} \quad \forall \; j \in J
\]

\[
\sum_{i} \mu_{b}^{i} = 1 \quad \forall \; b \in \{1,2, ..., B\}
\]

\[
\sum_{b} \mu_{b}^{i} \leq B \hat{y}_{i} \quad \forall \; i \in I
\]

\[
\sum_{j} X_{b ij} \leq L_{1}\mu_{b}^{i} \quad \forall \; i \in I, b \in \{1,2, ..., B\}
\]

\[
\sum_{i} X_{b ij} \leq L_{2}\hat{\alpha}_{j} \quad \forall \; j \in J, b \in \{1,2, ..., B\}
\]

\[
\sum_{i} \sum_{j} T_{ij} X_{b ij} \leq R_{max} \quad \forall \; b \in \{1,2, ..., B\}
\]

\[
q_{p} = \sum_{i} \delta_{p}^{i} d_{i}^{p} \quad \forall \; i, p \in I
\]

\[
d_{i} = \sum_{s=1}^{S_{i}} z_{s}^{i} d_{s}^{i} \quad \forall \; i \in I
\]

\[
\sum_{s=1}^{S_{i}} z_{s}^{i} = 1 \quad \forall \; i \in I
\]

\[
\sum_{s=2}^{S_{i}} z_{s}^{i} \leq \Gamma \quad \forall \; i \in I
\]

\[
z_{s}, \mu_{b}^{i} \in \{0,1\} \quad \forall \; i, j, s, b
\]
The purpose of solving the above subproblem is to find a demand vector which creates “trouble” with the current pick-up and shelter location plan. More specifically, the objective is to find a demand vector \( d \) that maximizes the minimum evacuation time for the given pick-up and shelter locations. If the optimal objective value of above subproblem is less than \( \xi \) (i.e., the objective value of master problem), then \((\hat{y}, \hat{\alpha})\) is the optimal pick-up and shelter location plan to the TS-RPSL problem. Otherwise, an improved location plan can be obtained by expanding the set of demand vectors \( \bar{D} \cup \{d^*\} \), where \( d^* \) is the solution to the above subproblem.

Because the number of demand vectors in the uncertainty set \( D \) is finite, the cutting plane algorithm terminates after a finite number of iterations. Below is the formal solution procedure of the cutting plane algorithm outlined above.

**Cutting Plane Algorithm:**

1. **Step 0:** Choose an initial demand vector \( d_1 \in D \). Set \( n = 1 \) and \( \bar{D} = \{d_1\} \).

2. **Step 1:** Solve the master problem (MP) with a subset of demand vectors \( \bar{D} \) and let \((\hat{y}^n, \hat{\alpha}^n, \hat{\delta}^n)\) denote the resulting optimal solution with objective function value as \( \xi \).

3. **Step 2:** Solve the subproblem associated with the pick-up location \( \hat{y}^n \) and shelter location \( \hat{\alpha}^n \) (Section 5.5.2) and let \( d^*_n \) denote the optimal solution and \( \hat{W}^n \) denote the corresponding objective value.

4. **Step 3:** If \( \hat{W}^n \leq \xi \), stop and \((\hat{y}^n, \hat{\alpha}^n, \hat{\delta}^n)\) is an optimal solution to the TS-RPSL problem. Otherwise, set \( \bar{D} = \bar{D} \cup \{d^*_n\}, n = n + 1 \) and go to step 1.

Furthermore, once the optimal pick-up \((\hat{y})\) and shelter locations \((\hat{\alpha})\) are obtained for the above TS-RSPL problem, the second-stage variables for any particular realized
demand ($\bar{d}$) can be obtained by solving the following minimization problem (MinET).

This problem can be solved directly and the solution is very easy to obtain.

MinET

\[
\min_{x,k,q,u} \sum_b \sum_i \sum_j T_{ij} x_{bij}
\]

s.t.

\[
q_p = \sum_i \delta^i_p \bar{d}^i, \quad \forall i, p \in I
\]

\[
\mu^i_b \in \{0,1\}, \quad \forall i, b
\]

\[
x_{bij} \text{ integer}
\]

and (5.38) – (5.44)

5.4.2 Solving Subproblem

Obtaining the solution to the subproblem is the most computationally difficult part of the proposed algorithm. The objective of SP model has a max-min structure which makes it difficult to solve. There are two levels in the SP formulation and it is not possible to obtain an equivalent one level formulation. Although it is difficult to calculate the objective function value itself, it is easier to obtain an approximation on the lower bound of the objective for the subproblem. More specifically, we make use of the dual problem associated with the subproblem to closely approximate the value of objective function. Since, subproblem is an integer program, we use the dual of the LP relaxation of the subproblem.

In the current subproblem formulation, allocation of buses to pick-up points, i.e., $\mu$, and the number of trips made by each bus, i.e., $x$, are both integer variables. Therefore, instead of writing the dual of the LP relaxation of the current subproblem, we will reformulate the subproblem such that the relaxation of integer constraints is minimized.
For given pick-up and shelter locations, and a known demand vector ($\bar{d}$), the problem to compute the minimum evacuation time (MinET) is re-formulated as:

**MinET-II**

$$
\min_{X,K} \sum_i \sum_j T_{ij}X_{ij}
$$

s.t.

$$
\begin{align*}
\sum_j \beta X_{ij} & \geq q_i \quad \forall \ i \in I \quad (5.50) \\
\sum_i \beta X_{ij} & \leq K_j \quad \forall \ j \in J \quad (5.51) \\
\sum_j X_{ij} & \leq L_1 \bar{y}_i \quad \forall \ i \in I \quad (5.52) \\
\sum_i X_{ij} & \leq L_2 \bar{a}_j \quad \forall \ j \in J \quad (5.53) \\
q_p = \sum_i \delta_p \bar{d}_i & \quad \forall \ i, p \in I \quad (5.54) \\
\sum_i \sum_j T_{ij}X_{ij} & \leq B \cdot R_{max} \quad (5.55)
\end{align*}
$$

where $X_{ij}$ is the number of trips from pick-up location $i$ to shelter $j$. The objective is to minimize the evacuation time. Constraint (5.50) ensures that all the evacuees at each pick-up location can be transported by the total trips. Constraint (5.51) requires the available capacity of public shelter to be greater than the total number of evacuees transported to that particular shelter. Constraint (5.52) specifies that the trips can be made from a location only when it is assigned as pick-up point. Constraint (5.53) specifies that trips are made to only open shelters. Constraint (5.54) estimates the total number of evacuees assembling at each pick-up location. Finally, constraint (5.55) imposes an upper bound on the average running time of each bus, where $B$ is the total number of available buses. Since the objective here is to find the minimum evacuation time, we do not exactly need the specific allocation of each bus. The information about
total number of trips between pick-up locations and shelter is sufficient in this case, in order to calculate the minimum evacuation time.

Now, the dual of the above LP relaxation of above problem can be written as:

\[
\text{D-MinET-II} \\
\text{max } \sum_p \sigma_p \left( \sum_i \delta_p^i d^i \right) - \sum_i L_1 \phi_i \hat{y}_i - \sum_j L_2 \theta_j \hat{a}_i - \omega BR_{max} \\
\text{s.t. } \\
\beta \pi_l - \beta \nu_j - \phi_i - \theta_j - \omega T_{ij} \leq T_{ij} \quad \forall \ i \in I, j \in J \quad (5.56) \\
\sigma_i - \pi_i \leq 0 \quad \forall \ i \in I \quad (5.57) \\
\nu_j \leq 0 \quad \forall \ j \in J \quad (5.58) \\
\pi_i, \theta_j, \phi_i, \omega \geq 0 \text{ and } \sigma_i \text{ unrestricted } \forall \ i \in I, j \in J \\
\text{where, } \pi_i, \nu_j, \phi_i, \theta_j, \sigma_i, \omega \text{ are the dual variable associated with the equations (5.50) – (5.55) respectively.}
\]

Now, the dual problem of the relaxed subproblem (D-RSP) can thus be written as below:

\[
\text{D-RSP} \\
\text{max}_{x,k,q,d,z} \sum_p \sigma_p \left( \sum_i \hat{d}_p^i d^i \right) - \sum_i L_1 \phi_i \hat{y}_i - \sum_j L_2 \theta_j \hat{a}_i - \omega BR_{max} \\
\text{s.t. } \\
\beta \pi_l - \beta \nu_j - \phi_i - \theta_j - \omega T_{ij} \leq T_{ij} \quad \forall \ i \in I, j \in J \quad (5.59) \\
\sigma_i - \pi_i \leq 0 \quad \forall \ i \in I \quad (5.60) \\
\nu_j \leq 0 \quad \forall \ j \in J \quad (5.61) \\
d^i = \sum_{s=1}^{s} z_s^i d_s^i \quad \forall \ i \in I \quad (5.62) \\
\sum_{s=1}^{s} z_s^i = 1 \quad \forall \ i \in I \quad (5.63)
\[ \sum_{i} \sum_{s=2}^{S_i} z_{is}^i \leq \Gamma \quad \forall \ i \in I \]

\[ z_{is}^i \in \{ 0, 1 \} \quad \forall \ i, s \]

\[ \pi_i, \theta_j, \phi_i, \omega \geq 0 \text{ and } \sigma_i \text{ unrestricted} \quad \forall \ i \in I, \ j \in J \]

Note that, the max-min formulation for subproblem is converted to max-max formulation using the dual of the relaxed subproblem. The above D-RSP model is a mixed-integer nonlinear program (MINLP), which is easier to solve and can be solved directly using DICOPT, a framework for solving MINLP (e.g., Grossmann et al., 2008). By solving D-RSP problem, we obtain a maximum objective function and a demand vector corresponding to that maximum value. Note that, the solution to D-RSP is a lower bound on the objective function value of the subproblem). In order to calculate the minimum evacuation time corresponding to obtained demand vector, we solve the MinET problem. This minimum evacuation time value is then compared with the current objective value of the master problem to check if we need to include this obtained demand vector into the set or not.

Relaxing the integer constraints and then computing the dual objective value can produce weak bound for the original problem. Therefore after solving the D-RSP, and obtaining an corresponding minimum evacuation time less than the current MP objective does not guarantee that there does not exist any other demand vector that can lead to a higher minimum evacuation time. Thus, in order to be on the safer side, a sampling procedure is done to ensure a good solution. After solving the D-RSP problem, if we get a demand vector for which the minimum evacuation time is higher than the objective of master problem, we include that demand vector in the set and go to Step 2 of the cutting plane scheme. Otherwise, 100 demand vectors are randomly
generated from the uncertainty set. For each sampled demand vector, MinET problem is solved to calculate the minimum evacuation time. If the minimum evacuation time for any of the sampled demand vector exceeds the objective of master problem, this particular demand vector is included in the set. If none of the 100 random demand vectors produces minimum evacuation time larger than MP objective value, we terminate the cutting plane algorithm.

5.5 Numerical Example

This section presents results from solving the proposed model on the Sioux Falls network (LeBlanc et al., 1975). The cutting plane algorithm is implemented in GAMS (Brooke et al., 1992) where the master problem (MP) and dual of the relaxed subproblem (D-RSP) are solved using CPLEX (CPLEX, 2008) and DICOPT (Grossmann et al., 2008) respectively.

Figure 5-1 displays the underlying network in which we assume that nodes 1-12, 16 and 18 are the demand points and eight potential sites to locate shelter in the network as nodes 13-15 and 20-24. Out of the eight potential sites, it is assumed that a maximum of six sites can be opened as shelters. It is further assumed that the planning agency has estimated three possible demand values at each demand point, i.e., nominal, low and high. The nominal demand is reported in Table 5-1 and the other two demand values, low and high, are generated by multiplying the nominal demand by a uniformly-distributed number between (0.5, 1) and (1.5, 2). It is further assumed that there are 10 buses available for evacuation and each has a capacity of 30. The maximum allowable running time of each bus is 180 min and the maximum allowable walking time from a demand point to a pick-up point is 5 min.
Figure 5-1. Sioux Falls network for demand points and potential shelter locations
Table 5-1. Demand values at each demand point

<table>
<thead>
<tr>
<th>Demand Points</th>
<th>Nominal Demand</th>
<th>Low Demand</th>
<th>High Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 1</td>
<td>50.00</td>
<td>29.29</td>
<td>78.27</td>
</tr>
<tr>
<td>Node 2</td>
<td>42.00</td>
<td>38.71</td>
<td>76.43</td>
</tr>
<tr>
<td>Node 3</td>
<td>40.00</td>
<td>31.01</td>
<td>63.19</td>
</tr>
<tr>
<td>Node 4</td>
<td>46.00</td>
<td>29.93</td>
<td>74.75</td>
</tr>
<tr>
<td>Node 5</td>
<td>38.00</td>
<td>24.55</td>
<td>69.71</td>
</tr>
<tr>
<td>Node 6</td>
<td>50.00</td>
<td>30.60</td>
<td>85.88</td>
</tr>
<tr>
<td>Node 7</td>
<td>34.00</td>
<td>22.95</td>
<td>57.11</td>
</tr>
<tr>
<td>Node 8</td>
<td>44.00</td>
<td>40.84</td>
<td>73.73</td>
</tr>
<tr>
<td>Node 9</td>
<td>44.00</td>
<td>23.48</td>
<td>68.89</td>
</tr>
<tr>
<td>Node 10</td>
<td>52.00</td>
<td>39.00</td>
<td>81.90</td>
</tr>
<tr>
<td>Node 11</td>
<td>48.00</td>
<td>47.95</td>
<td>86.14</td>
</tr>
<tr>
<td>Node 12</td>
<td>40.00</td>
<td>31.57</td>
<td>76.62</td>
</tr>
<tr>
<td>Node 16</td>
<td>36.00</td>
<td>35.84</td>
<td>58.15</td>
</tr>
<tr>
<td>Node 18</td>
<td>30.00</td>
<td>26.43</td>
<td>54.98</td>
</tr>
<tr>
<td>Total Demand</td>
<td>594.00</td>
<td>452.15</td>
<td>1005.75</td>
</tr>
</tbody>
</table>

Table 5-2 presents the optimal pick-up locations and shelter locations for the nominal, one-stage robust, two-stage robust and worst-case plans. The one-stage robust and two-stage robust plans are obtained using the degree of pessimism $\Gamma = 3$ while the nominal plan is obtained with $\Gamma = 0$ (deterministic demand with nominal value). The worst-case plan is the optimal plan against deterministic high demand at each demand point. It is equivalent, in this particular example, to solving the proposed model with the degree of pessimism, $\Gamma = 14$. 

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<table>
<thead>
<tr>
<th></th>
<th>Nominal Plan</th>
<th>One-Stage Robust Plan</th>
<th>Two-Stage Robust Plan</th>
<th>Worst-Case Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick-up Locations</td>
<td>Node 3</td>
<td>Node 6</td>
<td>Node 10</td>
<td>Node 12</td>
</tr>
<tr>
<td>Shelter Locations</td>
<td>Node 13</td>
<td>Node 14</td>
<td>Node 15</td>
<td>Node 15</td>
</tr>
<tr>
<td></td>
<td>Node 14</td>
<td>Node 9</td>
<td>Node 8</td>
<td>Node 18</td>
</tr>
<tr>
<td></td>
<td>Node 20</td>
<td>Node 11</td>
<td>Node 10</td>
<td>Node 11</td>
</tr>
<tr>
<td></td>
<td>Node 18</td>
<td>Node 18</td>
<td>Node 24</td>
<td>Node 18</td>
</tr>
<tr>
<td>Total Evacuation Time (mins)</td>
<td>765.00</td>
<td>1150.00</td>
<td>965.00</td>
<td>1260.00</td>
</tr>
</tbody>
</table>
The total evacuation time reported in Table 5-2 is the minimum evacuation time for nominal, one-stage robust and worst-case plan, whereas for the two-stage robust plan, it is the maximum value of minimum evacuation time among all the realized demand vectors. We can observe the difference in total evacuation time of the different plans and the fact that two-stage plan produce less conservative estimate as compared to the one-stage robust plan. It can be noted that for an degree of pessimism, $\Gamma = 3$, the two-stage robust plan result in 16% or more reduction in total evacuation time compared to the one-stage robust plan. Moreover, it has already been demonstrated that even though the nominal plan results in a lot of reduction in total evacuation time, it achieves very less reliability in terms of meeting the demand.

Next, in order to further demonstrate the flexibility of the two-stage robust plan compared to the one-stage robust plan, 1000 travel demand vectors are randomly generated from the uncertainty set for degree of pessimism, $\Gamma = 3$. For the one-stage robust approach, the total evacuation time will be same for all the generated demand vectors as reported in Table 5-2. On the other hand, according to the two-stage robust plan, we have different allocation of bus trips and shelter capacity estimate corresponding to each demand vector. Therefore, for the two-stage plan, the MinET problem is solved for each sampled demand vector, to compute the corresponding total evacuation time. Figure 5-2 shows the plot of the total minimum evacuation time for the sampled demand vectors corresponding to the one-stage and two-stage robust plans. It can be noted that the total evacuation time in the case of two-stage robust plan is always less than the evacuation time for one-stage plan. In addition, since the bus
allocation and shelter capacity estimate in two-stage robust plan is corresponding to the particular demand vector, the reliability of two-stage robust plan will always be 100%.

![Figure 5-2](image)

Figure 5-2. Plot of total evacuation time for sampled demand vectors

Table 5-3 compares the one-stage (the minimum evacuation time) and two-stage (the maximum of the minimum evacuation time) robust plans with varying the degree of pessimism. In general, the total evacuation time estimate increases with increase in the degree of pessimism, \( \Gamma \). As previously mentioned, \( \Gamma \) controls the degree of conservatism in the solution. An appropriate value of degree of pessimism can be chosen depending on the decision makers' attitude towards risk.
Table 5-3. Evacuation time of one-stage and two-stage robust plans with varying degree of pessimism

<table>
<thead>
<tr>
<th>Degree of Pessimism ($\Gamma$)</th>
<th>Total Evacuation Time (min)</th>
<th>One-Stage Robust Plan</th>
<th>Two-Stage Robust Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>915.00</td>
<td>820.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1055.00</td>
<td>900.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1150.00</td>
<td>965.00</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1175.00</td>
<td>1010.00</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1175.00</td>
<td>1055.00</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1175.00</td>
<td>1095.00</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1180.00</td>
<td>1120.00</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1180.00</td>
<td>1150.00</td>
<td></td>
</tr>
</tbody>
</table>

5.6 Summary

In this chapter, an integrated approach to model public shelters and pick-up locations for transit-based evacuation planning under demand uncertainty is proposed. The model is formulated as a mixed integer program to determine the optimal shelter locations, pick-up locations, allocation of available transit vehicles and capacities of public shelters. Two approaches, robust (single-stage) optimization and two-stage robust optimization are presented. The first approach i.e., single-stage robust optimization, determines all the decision variables before the realization of uncertain demand and constructs a solution that is feasible for all the possible realization of demands. In contrast, in the two-stage robust optimization approach, some of the decision variables are determined before the uncertain demands are realized while the other are decided once the realized demands are known. A cutting plane scheme is proposed to solve both the approaches.

Numerical results from Sioux Falls network demonstrate that the robust plans (one-stage and two-stage) yields better and reliable solutions. In addition, it was
demonstrated that two-stage robust optimization approach offers more flexibility to the decision maker than the single-stage approach and produce less conservative solutions. However, two-stage robust optimization problems are difficult to solve compared to one-stage robust optimization problems. It is believed that the two-stage robust optimization approach proposed in this chapter offers a new perspective on evacuation planning under demand uncertainty.

Our future work will involve demonstrating the effectiveness and usefulness of proposed model and solution algorithm to a realistic large-scale network. Proposed model can be further refined and extended to be more realistic and practical. For example, we assume that transit vehicles can make trips only from its assigned pick-up location. In future, better allocation of transit vehicle can be considered and possibly allowing for trip chaining. To model pick-up locations, we assume that evacuees will go the nearest pick-up location, which might not be the case in real-world. Evacuees may choose some other nearby pick-up location which is not necessarily closest, e.g., a more familiar location. In future, evacuee behavior in choosing the pick-up location can be studied. Although only demand uncertainty is considered, the proposed model framework is general enough to accommodate other types of uncertainties. In addition, model can be further extended to capture the time-dependent nature of evacuation into the framework.
CHAPTER 6
CONCLUSION

This dissertation develops robust optimization models for effective and reliable transit-based evacuation planning in order to evacuate transit-dependent population during emergency situations. Two critical decision-making components of transit evacuation planning are studied, namely, pick-up locations (staging areas) and public shelter locations. The proposed models take into account the demand uncertainty, which refers to the uncertainty in the number of transit-dependent evacuees. In particular, three models are proposed in this dissertation.

The first model focuses on transit evacuation planning on a regional level, with transit agencies in different counties performing their own evacuation. The proposed model is to determine the optimal locations of public shelters and their capacities, from a given set of potential sites for transit operations under demand uncertainty. The model is formulated as mathematical program with complementarity constraints and is solved by a cutting-plane scheme. A numerical example on the Sioux Falls network demonstrates that robust plans are able to achieve nearly the same level of performance with a significant lower cost as compared to a conservative plan, which assumes the highest demand of each origin node.

Next, a decision-support model is developed for a local level transit-based evacuation planning under demand uncertainty. A robust optimization model is proposed to locate the pick-up points for evacuees to assemble, and allocate available transit vehicles to transport the assembled evacuees between the pick-up locations and different public shelters. For this model, the locations of public shelters are assumed to be given and known. The model is formulated as a mixed integer linear program and is
solved via a cutting-plane scheme. The numerical results based on the Sioux Falls network demonstrates that the robust plan yields lower total evacuation time and is reliable in serving the realized evacuee demand.

In the last model, an integrated approach to model pick-up and shelter locations simultaneously for transit evacuation planning under demand uncertainty is considered. The developed model determines the optimal shelter locations, pick-up locations, allocation of given transit vehicles and capacities of public shelters. Two approaches, single-stage and two-stage robust optimization are proposed. The former is believed to produce conservative solutions, while the latter enables decision makers to fully take advantage of the revealed information to make less conservative decisions. In a two-stage robust optimization approach, the whole decision process is decomposed into two stages. In the first stage, optimal locations of pick-up points and public shelters are determined and once the realized demands are known, the allocation of given transit vehicles and shelter capacities are decided in the second-stage. A cutting plane scheme is used to solve both proposed approaches and results from the Sioux Falls network demonstrate that the robust plans (one-stage and two-stage) yield better and reliable solutions. In addition, the two-stage robust optimization approach offers more flexibility to the decision maker than the single-stage approach and produces less conservative solutions.

In order to reduce the impact of disasters in the future, planners need to devote at least as much attention to transit-based evacuation as to auto-based evacuation in emergency response plans, and develop ways to optimize the use of available resources during emergencies. In addition, it is important to understand why many
people ignore evacuation orders, e.g., some face logistical or transportation problem to evacuate, some had nowhere to go and are fearful of shelter conditions etc. Addressing these kinds of issues in an emergency response plan, pertaining to especially transit-dependent residents can increase evacuation rates and decrease damage to the society.

To our best knowledge, no analytical approach is available in the literature that determines the optimal shelter locations and transit pick-up locations during transit evacuation. The proposed models are very useful to planners, transit providers and emergency management officials for a better and effective transit evacuation planning. For emergency management and transit management agencies, how to efficiently operate the transit system while leaving no citizens behind is a key question. The results from this research provide strategies for efficient transit operation that address these issues.

The proposed models have the capability to be applied in real-world evacuation situations. In the future, the proposed models can be further refined and extended, to be more realistic and practical. Although only demand uncertainty is incorporated in the proposed models, the proposed framework is general enough to accommodate other types of uncertainty as well. Our future work may expand the current models to address a variety of uncertainties from both the demand side and supply side. Another future direction is to extend the models to capture the time-dependent nature of evacuation and incorporate delays and waiting times for evacuees to get to the pick-up locations. Future research may also examine the possibility of integrating both auto-based and the transit-based aspects of evacuation.
In summary, this dissertation develops transit-based evacuation plans to evacuate those who depend on public transportation to safe areas reliably and as fast as possible during an emergency. Optimization models are developed for transit-based evacuation planning which are more efficient and reliable in the face of demand uncertainty. The proposed modeling frameworks are feasible and promising for the robust and effective transit operations during evacuation.


Jenkins, B. M., & Winslow, F. E. (2003). Saving City Lifelines: Lessons Learned in the 9-11 Terrorist Attacks. Mineta Transportation Institute, San Jose University.


BIOGRAPHICAL SKETCH

Ashish Kulshrestha was born in India in 1985. He studied civil engineering at Indian Institute of Technology (IIT) Delhi, India and received his bachelor's degree in 2006. Afterwards he joined the University of Florida, Gainesville for the Ph.D. program in transportation engineering. During his Ph.D. study at the Transportation Research Center in University of Florida, Ashish Kulshrestha was a research assistant under the supervision of Dr. Yafeng Yin and worked on various projects.

Ashish Kulshrestha has co-authored six papers in transportation journals and conference proceedings and made presentations at various conferences. His primary research interests include transportation system modeling and optimization, managed lane operations, traffic control and operations, and travel demand modeling. He received his Doctor of Philosophy degree in December 2011.