MODELING THE IMPACT OF INTERNET TECHNOLOGY ON MARKETING

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2002
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by

Kutsal Dogan
To my family
ACKNOWLEDGMENTS

The very first thing I was told when I decided to embark on this journey and pursue a Ph.D. degree was that it was going to be a learning experience. It has really been a memorable journey.

I would like to thank all the people that I came across during this journey. Specifically, I thank Dr. Hsing Kenny Cheng, the chairman of my dissertation committee, for his time, guidance, support and remarkable mentorship throughout this research. I greatly appreciate the guidance and wisdom of Dr. Gary J. Koehler, the cochairman of my dissertation committee. I also thank Drs. S. Selcuk Erenguc, Steven M. Shugan and Asoo J. Vakharia, the members of my dissertation committee, for serving on my committee and their help.

I wish to thank my wife, Bahar, for her love, friendship and unbelievable optimism that pushed me forward over the years. I would also like to thank my beloved parents and sister for their unconditional love and support.
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Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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This dissertation, to the best of our knowledge, is among the first to address Internet coupons. The aim is to develop an analytical framework to investigate how Internet coupons can be used to target consumers and efficiently price discriminate to improve a firm’s profit.

First, Salop’s spatial model is modified to represent conventional paper coupons. The model created in this study for conventional paper coupons contributes to the coupons literature in general and has distinct features that separate it from the previous models proposed in the literature. A firm’s problem to determine the optimal price, the optimal coupon face value, and the optimal coupon distribution intensity (fraction of consumers who get a coupon) with a conventional paper coupon is solved.

Second, we look at Internet coupons and examine a fixed face value coupon model first. In an alternative model, a different kind of couponing model with a distinct
feature, changing face value, is introduced. Without the flexibility of the Internet, firms cannot change the face value of their coupon offerings according to individual consumers’ preferences and shopping behavior. However the Internet provides this flexibility to marketers. When we have detailed information regarding consumers’ shopping behavior and preferences, we can also estimate their willingness to pay to some extent. With traditional paper coupons a firm is limited to a very small number of different coupons. In practice, a conventional firm generally uses one coupon with the same face value for all consumers. Online marketing allows firms to target their consumers in ways that were not feasible before. Therefore, a firm can effectively target consumers and decide who should be given what coupon.

Finally, the use of coupons, conventional vs. Internet, in a competitive setting is analyzed. A firm’s decision to use coupons as a strategic tool in a competitive environment is examined. We look at the cases where two firms compete with each other and use conventional paper or Internet coupons. We calculate the equilibrium profit and market share figures for the firms for alternative couponing strategies and find the general conditions under which one is advantageous to the other.
CHAPTER 1
INTRODUCTION

1.1 Internet Coupons

The Internet and the relevant technologies are affecting all functional areas of organizations including marketing. Within marketing, advertising on the Internet has increased rapidly in recent years. It appears that the Internet will influence sales promotions, especially couponing, tremendously in the near future as much as it had influenced advertising since 1996. In 1999, Internet advertising formed 3 percent of the US advertising market. With the advent and increased availability of broadband networking, Internet advertising is expected to flourish. Morgan Stanley Dean Witter (2000) estimates that Internet advertising will increase its share up to 14 percent by 2005. Similar to what Internet advertising was in 1996, currently, Internet couponing accounts for less than 0.5 percent of the total U.S. couponing market compared to free standing inserts (FSIs), which have the largest share among all coupon drop methods with 76 percent. This is a miniscule market share. However, it is expected that Internet couponing will achieve a respectable market share among all coupon drop methods in the near future (Morgan Stanley Dean Witter 2000). There are major differences between conventional forms of couponing and Internet couponing. These differences have significant effects on firms, consumers and the overall market microstructure. This study explores these differences and estimates their effects using analytical microeconomic models. It is shown that treating Internet coupons the same as conventional paper coupons will have unexpected consequences that will defeat one of the most
important purposes of coupons: price discrimination. To benefit from this new form of providing coupons firms need to understand its advantages and disadvantages. How much firms will benefit from Internet coupons depend on how they will be able to tailor their couponing strategies to suit Internet coupons.

1.2 Advantages and Disadvantages of Internet Coupons

Internet couponing has some advantages over the classical mass-media coupon drop methods. The Association of Coupon Professionals (ACP) lists the benefits for marketers in its 2001 guide to Internet coupons as follows: new capabilities for one-to-one marketing; upscale, educated audience of Internet coupons; potential new coupon users; possibility of links to company websites for further information; ease of tracking and measurability; relationship building with subscription to marketer services; ability to integrate coupons with online activities and the flexibility of the entire couponing process (Association of Coupon Professionals 2001). In addition to these benefits, coupon distribution costs and lead-time for coupon drop are extremely low compared to FSIs provided in newspapers and other coupon drop methods. Furthermore, the redemption rates for Internet coupons are reported to reach up to 20 percent whereas conventional paper coupons provided as FSIs achieve a mere 1.3 percent redemption rate (Planet U 2001, Santella 2001). The most important of all is that customer profiling through monitoring consumers’ online shopping behavior and preferences and self reported demographics allow firms to better target their coupons to right consumers. This will further facilitate the customization of coupons and other promotional offers at the individual consumer level. Online consumer promotions are a revolutionary tactic for obtaining a one-to-one marketing relationship with consumers. Online promotions can play an instrumental role in attracting consumers and developing an accurate marketing
program that is not wasted on those outside the target market. Even with conventional paper coupons, targeting is possible to a certain degree. However, the cost of customization is usually too prohibitive to tailor an offer at individual level.

Despite its overwhelming advantages, Internet coupons have some disadvantages for marketers that demand attention if they are to command a better market share or to replace paper coupons altogether. There are alternative ways of providing Internet coupons that are reviewed in detail in Chapter 2. One way to distribute coupons over the Internet is print-at-home coupons that consumers can view and print in the convenience of their homes. Coupons provided this way are prone to fraud and counterfeiting. The March 2001 report of Coupon Information Corporation, a nonprofit association of consumer product manufacturers trying to eliminate coupon misredemption and fraud, voices the concerns regarding the security risks of print-at-home coupons and shows how a 14-year old kid can manipulate face values, expiration dates and terms of coupons and defeat the very purpose of coupons (Coupon Information Corporation 2001). Even though other forms of Internet coupon drop can eliminate most of these concerns, this report shows the common perception among big consumer packaged goods manufacturers against Internet coupons, print-at-home or otherwise. This misconception might be the reason that holds Internet couponing back despite its many advantages. We believe the resistance will be eradicated in time with better communication between manufacturers/retailers and Internet couponing firms because the benefits are too great to pass on.

Besides the benefits for coupon providers, Internet coupons have some benefits for consumers as well. The benefits given in the Association of Coupon Professionals
guide to Internet coupons include ease of use, convenience, relevance of coupons, presentation, richness of information and additional incentives that can be garnered. The ease of use and high level of convenience for Internet coupons diminish the cost of using coupons for consumers, in some case to a negligible level. This is obviously good for consumers. However, this may or may not be beneficial to coupon providers depending on couponing program they implement.

1.3 Research Problem

One of the many objectives of couponing is to price discriminate between price-sensitive and price-insensitive consumers. Other possible objectives of coupons are reviewed in detail in Chapter 3. Narasimhan (1984) showed evidence regarding the price discrimination effects of coupons. As mentioned earlier, Internet coupons achieve a very high redemption rate and a part of the reason is their ease of use. A November 1999 NPD online research found that 52 percent of consumers who used a coupon in the previous month obtained an online coupon and 78 percent of these consumers actually used them (NPD Group 1999). This may be good news to coupon providers given that only targeted consumers see their Internet coupons. If their targeted coupons become mass-media coupons, through unlawful duplication or incorrect distribution strategy by providers, the results may or may not be what they were intended to be. If this occurs, the price discrimination will not be possible.

This study, to the best of our knowledge, is among the first to address Internet coupons. The aim is to develop an analytical framework to investigate how Internet coupons can be used to target consumers and efficiently price discriminate to improve a firm’s profit. The use of Internet coupons and conventional coupons as a strategic tool to gain competitive advantage is also analyzed using the models developed in a duopolistic
setting. We extend Salop’s spatial model (Salop 1979) to address issues related to both conventional and Internet coupons.

First, in Chapter 4, Salop’s model is modified to represent conventional paper coupons. A firm that provides a form of traditional paper coupons that are mass distributed without any targeting is called “conventional firm,” “paper firm” or “paper coupon firm” throughout this study. These terms are used interchangeably. Even though this is not the first time a spatial model is applied to coupons, it is the first time that Salop’s model is used in the context of coupons. The model created in this study contributes to the coupons literature in general and has distinct features that separate it from the previous models proposed in the literature. In the literature, an individual’s decision to use a coupon is modeled (see Narasimhan 1984); however, this process has not been incorporated in a firm’s decision process. This study incorporates consumers’ decision process in the firm’s decision process through the use of utility maximization principle for each individual consumer. A firm’s problem to determine the optimal price, the optimal coupon face value, and the optimal coupon distribution intensity (fraction of consumers who get a coupon) of the product with a conventional paper coupon is solved in Chapter 4. For the optimal levels of these three decision variables, resulting coupon redemption rate, market share and profit figures are calculated.

In Chapter 5, a monopolistic seller’s decision to use Internet coupons is examined. The model proposed in Chapter 5 has one important feature: changing face value coupons. Without the flexibility of the Internet, firms cannot change the face value of their coupon offerings according to individual consumers’ preferences and shopping behavior. However the Internet provides this flexibility to marketers. When we have
detailed information regarding consumers’ shopping behavior and preferences we can also estimate their willingness to pay to some extent. With the help of coupons a monopolist can charge higher prices to some consumers and lower prices to others and segment consumers into many segments. However, with traditional paper coupons the firm is limited to a very small number of different coupons. In practice, a conventional firm generally uses one coupon with the same face value for all consumers. Online marketing allows firms to target their consumers in ways that were not feasible before. Therefore, the firm can effectively target those consumers who should be given a coupon. In the classic coupons literature a coupon’s face value is constant and the same for all consumers. With single face value Internet coupons, firms benefit from the low cost of coupon distribution on the Internet and the ability to target. However, they do not extract the benefits of one-to-one marketing completely. With the advent of tracking and personal communications technologies, firms can individually target each consumer and tailor its coupon according to individual consumers’ preferences and willingness to pay. Instead of one coupon, a separate coupon for each consumer can be issued for immediate online use. Online consumer behavior can be analyzed in order to determine consumers’ desire for a product. According to the information gathered this way the firm can then decide on the coupon and the terms of the coupon. Our study is interested only in the face value of the coupon. Other features such as the terms and the timing of the coupon are not dealt with. We assume that the coupon is given for one product and it is instantaneous as soon as the face value is determined. It is fair to state that the coupons distributed this way will not impose any cost of using.
In Chapter 6, we analyze the use of coupons, conventional paper vs. Internet, in a competitive setting. A firm’s decision to use coupons as a strategic tool in a competitive environment is examined. We look at the case in which two firms compete with each other and use no coupons, conventional paper coupons or Internet coupons. We calculate the equilibrium profit and market share figures for the firms for alternative couponing strategies and find the general conditions under which one technique is advantageous to the other. Finally, Chapter 7 concludes the work with the summary of major findings and a sketch of future research endeavors.
2.1 History of Coupons

In 1894, Asa Chandler bought a $2300 formula for the most popular soft drink in the world, Coca-Cola. He offered the first coupons in the form of handwritten tickets for a free trial of his new fountain drink. The next year, C.W. Post followed suit by distributing the first grocery coupon. Its value was one cent and was to be applied to the purchase of his new health cereal, Grape Nuts. During the Great Depression, coupons were an essential element in the American household to save money on the grocery bill each week. In the 1940s, grocery stores popped up across the country and followed the coupon example set by the neighborhood grocer.

The Nielson Coupon Clearing House shaped a new industry by becoming the first coupon redemption-clearing house in 1957. Eight years later, in 1965, one-half of all Americans redeemed coupons. In 1975, over 35 billion coupons were distributed as freestanding inserts (FSI) and 65 percent of U.S. households clipped coupons. In 1984, Catalina Marketing introduced electronic coupons. Companies saw the opportunity for different means of distribution, such as direct mail, in-store kiosks, at-shelf coupons, and various in-store promotions. The first Internet coupons came on the scene in 1995. According to CMS, Inc. research, in 2000, 330 billion coupons were distributed to American consumers; out of these 4.4 billion coupons were redeemed (a redemption rate of 1.3 percent) for savings of $3.3 billion (Santella 2001).
2.2 Types of Coupons

The type of coupon that holds the lion’s share of distribution is the FSI that is distributed in the Sunday paper each week. Local grocers generally offer a mid-week insert for their specific store to promote different specials. Direct mail is also a prominent method of distributing coupons. Val-Pak Direct Marketing Systems, Inc., the leader and originator of local cooperative direct mail, began its quest to provide savings to consumers by mail in 1968. The United States Postal Service reports that 89 percent of Americans read and/or look at the advertisements they receive in the mail. According to the data collected by the Direct Marketing Association, every dollar spent on direct mail advertising resulted in more than $10 in sales (Dalzell 1998).

Electronic couponing has also made quite an impact on the couponing front. The major contestants in that competition are Catalina Marketing Network, News America Marketing (formerly ActMedia), and InterAct Systems (acquired by the Sage Group). Catalina Marketing has incorporated its network of printers and PCs into 15,635 supermarkets nationwide to target customers at the point-of-purchase based on the items they purchase. News America Marketing provides over 50,000 stores with its at-shelf couponing system. The couponing devices are placed adjacent to the specific item for consumers to obtain as they are walking by the product. News America Marketing brags it has redemption rates of 17 percent and impulse purchase rates of 58 percent (News America Marketing 2002).

InterAct Systems provides yet another version of electronic couponing via in-store kiosks. The customer swipes his frequent shopper card at a terminal at the front of the store and in 60 seconds 40 targeted items related to the shopper’s purchase history appear on the screen. By touching the screen, the shopper can choose the items of
interest to him. The InterAct system automatically deducts the electronic coupons at the
cash register when the targeted items are purchased.

The latest multi-faceted entrant into the couponing arena is the Internet coupon.
The Internet coupon comes in just as many varieties as the traditional paper coupon.
There are four major players currently in the US Internet Couponing Market:
CoolSavings®.com, ValuPage® (owned by SuperMarkets Online®, Inc. which is a
subsidiary of Catalina Marketing Inc.), Planet U™ and Coupons Inc. (formerly
Storecoupon). There are other players including DirectCoupons.com, H.O.T.!
Coupons.com, CouponSurfer.com, MyCoupons.com, Save.com. However, the first four
firms represent the overwhelming majority of the market and four distinct ways to
distribute Internet coupons.

2.3 Brief Review of Coupons of Late

Traditionally, freestanding inserts (FSI) are the most far-reaching with a
distribution rate of 76 percent of the total coupon distribution. However, with a quick
decline among consumer packaged goods manufacturers, FSIs experienced a great
decline and currently attain a redemption rate of only 1.3 percent. The reason lies in the
targeted couponing campaigns via at-shelf coupon machines with a redemption rate of 17
percent and checkout coupons that average 9 percent redemption. Marketers begin to
know the purchase history of individual buyers and can target their promotions to
particular people. Charles Brown (marketing vice president at NCH Promotional
Services) states, “done correctly, these promotions offer more effective redemption,
enable marketers to reach their target at a specific time to increase trial, spark repeat
purchase or gain trial among a competitor’s user” (Thompson 1997, p. 35).
2.3.1 ValuPage®

Catalina Marketing Network® acted as a catalyst to spur manufacturers and retailers alike to rethink their conventional views of couponing. As the leader in electronic coupons (those printed at the point of purchase), Catalina saw an opportunity to quickly seize another niche of the marketplace by introducing its Internet coupon website ValuPage®. For more than three years SuperMarkets Online®, Inc. has been providing incentives to its online users through its ValuPage® Website. The website receives 2.6 million unique visitors each month and it is ranked as the number one home and food site on the Internet according to its Chris Linskey, General Manager and Senior Vice President for SuperMarkets Online®, Inc (PR Newswire 2001). It is also a top 75 Website most visited by women. A further 3.4 million shoppers (20 percent being AOL subscribers) are current members of the ValuPage® system and receive weekly emails, called ValuPageE-mailSM, indicating different promotional values.

To utilize the savings offered from ValuPage®, you may visit any site that features the ValuPage® banner or go directly to ValuPage® or Supermarkets Online®. You then enter your zip code and a list of participating retailers appears before you. There are more than 13,400 supermarkets across the country participating in the program. You print out the actual ValuPage®, a list of items offering savings, with the bar code printed at the top. Present your ValuPage® to the cashier at any point during checkout. When the bar code of an item you purchased matches that of an item on the ValuPage®, the Catalina computer prints the appropriate Web Buck®. Web Bucks® are a cash-like reward redeemable on your subsequent visit to the same supermarket chain to purchase anything you want (not just the items on the ValuPage®), with the exception of items
prohibited by state law, i.e., beer, liquor, and cigarettes. This encourages repeat shopping at the same chain, which is a great benefit for retailers (ValuPage® website). You must wait until the next visit to redeem your Web Bucks® because the consumer packaged goods industry is extremely hesitant about offering instant cash back rewards accessible over the Internet. The opportunity for fraudulent behavior is too prevalent. The issuance of the Web Bucks® after you have purchased the promoted product eliminates the possibility of duplicitous actions.

2.3.2 CoolSavings®

According to the Jupiter Media Metrix May 2001 report CoolSavings®.com is the number one coupon site on the Internet. It currently has 15 million users and 13.4 million households subscribe to its services (CoolSavings 2001). Upon conception in March 1997, CoolSavings®.com began offering numerous coupons online from nationally recognized retailers for travel, entertainment, restaurant chains, department stores, and toy stores.

CoolSavings®.com does not currently support purchasing online; however, it does offer links to the websites of particular merchants who sell merchandise online. It has a distinct method to offer savings to the consumer through a small application one must download. The application, Coupon Manager, enables the consumer to print selected coupons. Upon printing, the coupons will be downloaded to the Coupon Manager, which will print the coupon without the consumer ever seeing it. The coupon will also be taken off from the user’s list of coupons. Each coupon can be printed only once.
2.3.2 Planet U™

Planet U™ offers coupons to be selected online and then sent via U.S. mail or securely and electronically downloaded directly to a participating retailer’s point-of-sale (POS) system. These coupons are redeemed at the time of purchase when consumers swipe their frequent shopper cards at the POS system. As a third choice, consumers can go to the coupon providers’ website directly and redeem the coupons towards an online purchase. Their product is called U-pon, “the Internet coupon with intelligence,” a targeted Internet coupon. Since introduction of the U-pon in February 1998, the redemption rate of these highly targeted coupons has been a remarkable 20 percent (Planet U 1998).

The customer is asked to fill out a one-page registration sheet in order to obtain demographic information to be used more effectively to offer savings. Information is also gathered as the customer actually selects and redeems the offers. Planet U™ asks only for limited data, such as street and email address for the distribution of the coupons, but asks for as much as the customer is willing to give so as to customize the offerings more precisely.

Planet U™ has completely eliminated coupon fraud from entering the scenario by printing the coupon on copy-resistant security paper containing a watermark, a pantograph, and UPC EAN 128 codes that can be traced back to the household and transaction level. The pantograph will read “void” if copied. The U-pons are then mailed from a Planet U™ outlet or the manufacturer’s clearing house.
2.3.4 Coupons Inc.

Coupons Inc. offers targeted print-at-home coupons for in-store or online redemption through its secure, patent-pending PASSport technology. It provides software to be downloaded onto one’s browser as a toolbar to continually peruse available savings. The accessibility is revolutionary for the brand awareness of both Coupons Inc. and of the advertised products.

Manufacturers or retailers can use three ways to deliver coupons to consumers: 1) Bricks delivers coupons directly from website to consumers; 2) AdBricks delivers coupons on online banner ads; 3) eBricks delivers through targeted, opt-in e-mails. Its coupon printer software prints coupons that contain an encrypted 2 dimensional bar code (PDF417). Consumers never see the coupon on their screens since it is directly transferred to their printer. This, combined with a closed-loop redemption checking by the coupon provider’s clearing partner, guarantees a tight security measure against coupon fraud. For security, the coupon may not be viewed on the screen so as to prevent manipulation of the coupon (Dalton 1999). Currently the Coupons Inc site averages over 1.7 million visitors a month.

Internet couponing has taken quite a foothold on American consumers thus far. Once viewed as a novelty, the future of paperless couponing is simply around the virtual corner. Table 1 gives a summary of the four major online couponing firms. Each of the four firms has its own security mechanism to make sure coupon promotions are not only easy to handle but also secure. Once these security measures are communicated to security conscious coupon providers, Internet couponing is likely to increase its market share among all coupon drop methods. As mentioned earlier, theoretical research on coupons that applies to Internet couponing is very scant but the increase in the market
share will make the research in this area very valuable to coupon providers. In the next chapter, the detailed review of coupons literature related to our work is given.
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<th>Coupons Inc.</th>
<th>Planet U™</th>
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<td>Enter zip code, print out ValuPage® with savings up to $50</td>
<td>Use Coupon Manager software to “clip” and print selected coupons</td>
<td>Download software that is constantly searching for savings and delivers them to the one’s browser</td>
<td>Log onto partner, marketer, or retailer website and select coupons</td>
<td></td>
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<tr>
<td>Delivery</td>
<td>Present at checkout and receive Web Bucks® to be used on any subsequent grocery purchase</td>
<td>Use printed coupons for discounts on national brands of groceries, travel, entertainment, toys, restaurants, department stores</td>
<td>User peruses the GUI at his leisure, selects, and prints the desired coupons for use on numerous participating national brands</td>
<td>Coupons are sent via United States Mail, downloaded to one’s frequent shopper card or redeemed towards online shopping.</td>
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<tr>
<td>Security</td>
<td>Web Bucks® redeemed upon next visit; no cash rewards offered directly online</td>
<td>Coupons have expiration dates and may be printed only once; users never see the coupons.</td>
<td>Proprietary encryption portal system; coupon may not be viewed on screen; coupons contain an encrypted 2 dimensional bar code (PDF417) as well as a standard bar code.</td>
<td>Coupons printed on copy-resistant security paper with a watermark, pantograph, and UPC 128 codes able to trace back to households</td>
</tr>
<tr>
<td>Software Download</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Redemption Rates</td>
<td>15%</td>
<td>Undisclosed</td>
<td>Over 10%</td>
<td>20%</td>
</tr>
<tr>
<td>Targeted E-mail Coupons</td>
<td>Yes (ValuPageEmailSM)</td>
<td>Yes (E-mail Target Definition Service)</td>
<td>Yes (eBricks)</td>
<td>Yes</td>
</tr>
<tr>
<td>Banner Ad Coupons</td>
<td>Yes (ValuBannerSM)</td>
<td>No</td>
<td>Yes (adBricks)</td>
<td>Yes (Ad U-pon)</td>
</tr>
</tbody>
</table>
CHAPTER 3
LITERATURE REVIEW

Marketing literature on coupons is extremely broad. A lot of research has been devoted to examining the effectiveness of coupons in achieving objectives set forth by coupon providers. Blattberg and Neslin (1990) provide an excellent review of the literature on coupons up to 1990. Their framework classifies the objectives served by coupons under four general categories: “direct sales impact,” “retail trade related,” “integrate with advertising and other promotions” and “use as a strategic tool” (see Table 2). Our work is most closely related to “price discriminate,” “attracting brand switchers” and “reach the appropriate target group” objectives. In Chapters 4 and 5, we analyze coupons as price discrimination devices. We also compare the use of conventional and Internet coupons in terms of their use as a strategic tool. In Chapter 6 we look at coupons’ effect on brand switching.

Table 2 Couponing Objectives

<table>
<thead>
<tr>
<th>Direct Sales Impact</th>
<th>Retail Trade Related</th>
<th>Integrate with Advertising and Other Promotions</th>
<th>Use as a Strategic Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attract new triers</td>
<td>Gain in-store promotional support</td>
<td>Reinforce or enhance print media advertising</td>
<td>Preempt the competition</td>
</tr>
<tr>
<td>Attract brand switchers</td>
<td>Increase distribution</td>
<td>“Synergize” with other marketing instruments</td>
<td>Price discriminate</td>
</tr>
<tr>
<td>Increase category consumption</td>
<td>Motivate the sales force</td>
<td></td>
<td>Cushion a price increase</td>
</tr>
<tr>
<td>Maintain or enhance repeat purchase rates</td>
<td>More directly control retail price</td>
<td></td>
<td>Reach the appropriate target group</td>
</tr>
<tr>
<td>Defend market share</td>
<td></td>
<td></td>
<td>Gain trial for another product</td>
</tr>
</tbody>
</table>

Source: Blattberg and Neslin (1990), p. 269, Table 10.4
Blattberg and Neslin (1990) detail the research that provides theoretical and empirical evidence to each of the objectives given in Table 2. As for the price discrimination effects of coupons, Narasimhan (1984) provides theoretical and empirical evidence regarding the price discrimination benefits of coupons and shows that the consumers with lower opportunity cost for their time are most likely to use coupons. He finds empirical evidence that coupon users are more price sensitive. Babakus et al. (1988) also provide empirical evidence supporting the price sensitivity argument.

Narasimhan (1984) develops an individual consumer’s decision to use coupons and states that the trade-off between the value of coupons and the cost of using coupons determines which decision is made. He also shows that a certain segment of consumers can be charged a lower price with use of coupons, namely those who elect to use coupons. However, he does not look at the problem from a firm’s perspective. Our study analyzes a firm’s decision, taking into account consumers’ decision process. Others also found support for the price discrimination characteristic of coupons. White (1983) posits coupons as a price discrimination tool used by manufacturers to segment consumers into two classes: those who comparison-shop and those who do not. Levedahl (1986) supports the price discrimination objective of coupons. Houston and Howe (1985) provide a theoretical analysis of coupons as a price discrimination tool under monopolistic competition. In a recent paper, Nevo and Wolfram also find support for price discrimination models in oligopoly settings (Nevo and Wolfram 2001).

There is a body of research that looks at the firm’s decision process to use coupons as a strategic tool in a competitive environment. Caminal and Matutes (1990), Chiang (1995), Bester and Petrakis (1996), Shaffer and Zhang (1995) and most recently
Nevo and Wolfram (2001) analyze competitive effects of coupons. There are major differences between our work and this literature. Bester and Petrakis (1996) model coupon competition in a duopolistic market and calculate the optimal levels of price, coupon face value, and coupon distribution intensity. As their model looks only at the competition of conventional coupons, costs of using coupons is not included in the model. Caminal and Matutes (1990) look at a two-stage model where coupons are used to create lock-in effect in the second period. Our models do not consider repeat purchases as we assume each consumer is in the market for one product. Chiang (1995) endogenizes consumers’ decision to use coupons in the overall demand decision and bases this decision on consumers’ utility maximization to measure the impact of coupons on the category sales. The model developed in Shaffer and Zhang (1995) is closest to the models developed in this work. They develop a two-stage game theoretic model of pricing and coupon distribution to compare conventional mass-media coupon drop to paper coupons targeted based on consumers’ past purchases. They use a linear spatial model to compare two types of coupon drop methods. They find that coupon targeting is not necessarily a replacement for mass media distribution of coupons as the two methods work differently in segmenting the market. Their model differs from ours as they do not allow for different levels of mass media coupon drop and do not calculate the optimal level of mass media coupon drop intensity. Another important difference is that their model does not incorporate individual consumers’ coupon use decision. A constant fraction of consumer population is assumed to be coupon users and the rest are assumed to be nonusers. Therefore, the face value of the coupon and the cost of using the coupon
are not used to determine coupon users and nonusers. Our models allow us to determine if an individual is a coupon user if she receives a coupon with a certain face value.

The use of coupons as a targeting tool has also attracted some research. Alsop (1985), Stern (1985), Zeldis (1987) and Raphel (1988) give examples of coupon targeting through direct mail programs such as Donnelly Marketing’s Carol Wright program. Shaffer and Zhang (1995) state that coupons should be targeted to a competitive firm’s consumers to increase brand sales. With the advent of Internet tracking technologies, firms can now individually target each consumer and tailor their coupon according to the individual consumer’s preferences and willingness to pay. Instead of one coupon, separate coupons for each consumer can be issued for immediate online use. Kumar et al. (1998) construct an Internet coupon distribution system and the relevant systems to assure a smooth and secure couponing process. In a later paper, Rangachari et al. (1999) describe how a consumer’s online behavior can be analyzed in order to determine her desire for a product. According to the information gathered this way the firm can then decide on the coupon and the terms of the coupon.
CHAPTER 4
SPATIAL MODEL OF CONVENTIONAL PAPER COUPONS

The aim of this chapter is to develop an analytical framework to examine how conventional paper coupons are used to efficiently price discriminate to improve a firm’s profit. Salop’s unit circle model is used as a base model to represent conventional paper coupons (Salop, 1979). Next, we review Salop’s model.

4.1 Spatial Model of a Firm: Review of Salop’s Model

Consider a monopolistic market for a single product where the monopolist produces the product with constant returns to scale and at a constant marginal cost. We assume that the marginal cost of production is zero. We use the product characteristics approach to product differentiation introduced in Hotelling (1929) and developed by Lancaster (1975) and Salop (1979). The market is modeled here and in the rest of the paper as a unit circle (the total number of consumers is assumed to be constant and normalized to one) following the model given in Salop (1979). Consumers have a common reservation price for the product, \( r \). We assume that each consumer is in the market for at most a single quantity of the product and all consumers have perfect information about different market offerings and prices.

Consumers are assumed to have heterogeneous preference and are uniformly distributed along the unit circle according to their preferences for the product. Each consumer’s location on the unit circle represents her ideal product. Consumers incur a fit cost \( t \) per unit distance, as they fall away from their ideal product. The fit cost represents consumers’ dissatisfaction due to the difference between the characteristics of the product
that is supplied by the firm and the characteristics of consumers’ ideal product. A product is represented by a point on the unit circle. The monopolist sells the product at price \( p \). All of these assumptions are commonly made in the economics and marketing literature (see Chen et al. 2001, Dewan et al. 2000, Grossman and Shapiro 1984).

Figure 1 gives a visual representation of the unit circle model. In the figure the monopolist markets the product at point B. The marginal consumers who are distance \( d \) away from their ideal product determine the size of the market and they are located at two locations, point \( A \) and point \( C \).

These two consumers (\( A \) and \( C \)) are indifferent between buying and not buying and the following equation holds for them.

\[
0 = r p - d t
\]

Thus, \( d = \frac{r - p}{t} \). All consumers whose distance to their ideal product is less than \( d \) will buy the product at price \( p \) since \( r - p - d_t > 0 \) for all \( d_t < d \). That is, all consumers along the arc from \( A \) to \( C \) will have a positive net surplus. Since within distance \( d \) of point \( B \) everyone buys, the total market size of buyers is \( 2d \). If we let \( \Pi_{NC} \) denote the profit for
the no coupon (NC) case, the firm’s profit is $\Pi_{NC} = 2dp = 2\left(\frac{r-p}{t}\right)p$. The monopolist solves the following maximization problem. 

$$\max_p \left[ \Pi_{NC} \right] = \max_p \left[ 2\left(\frac{r-p}{t}\right)p \right].$$

First Order Condition (F.O.C.) for maximization is as follows: 

$$\frac{d\Pi_{NC}}{dp} = 2\left(\frac{r-2p^*}{t}\right) = 0.$$

Since $\frac{d^2\Pi_{NC}}{dp^2} = -\frac{4}{t} < 0$ for all $t > 0$, the second order condition for maximization is also met. Hence we find the solution as $p^* = \frac{r}{2}$ and $\Pi_{NC}^* = \frac{r^2}{2t}$. We have $d^* = \frac{r-p^*}{t} = \frac{r}{2t}$.

Let $k$ be the percentage of the market that is covered, or simply the market coverage. Then, $k^* = 2d^* = \frac{r}{t}$. When $r \geq t$, it is optimal for the monopolist to serve all consumers. If $r < t$, some consumers are not served at the equilibrium. We assume that $r < t$; therefore, the monopolist has an incentive to reduce prices for some consumers through coupons.

**4.2 Spatial Model of Conventional Paper Coupons**

In this section, we model a firm’s decision process to use traditional paper coupons. We solve for the optimal price, the optimal coupon face value, and the optimal coupon distribution intensity (fraction of consumers who get a coupon) of the product. For the optimal levels of these three decision variables, resulting coupon redemption rate, market share and profit figures are also calculated.

As we mentioned in Chapter 1, this is not the first time a spatial model is applied to coupons; however, this is the first time that Salop’s model is used in the context of coupons. The model created here adds to the extant coupons literature with its distinct
features, which also separate it from the other models developed earlier. In coupons literature, an individual’s decision to use a coupon is modeled (see Reibstein and Traver 1982, Narasimhan 1984, Chiang 1995); however, this process has not been incorporated in a firm’s decision process. This study combines individual consumer’s decision process with the decision process of a coupon proving firm by using the principle of utility maximization. Each individual consumer’s decision to become a coupon user is endogenous to our model. In earlier works, where a firm’s coupon use decision was analyzed, the decision to use coupons was included exogenously by assuming a certain fraction of consumers to be coupon users (see Shaffer and Zhang 1995). This fraction was assumed to be constant and did not depend on the face value of the coupon or the disutility the consumers have to incur in order to use coupons. Our model explicitly incorporates the face value and the disutility of using coupons. Therefore, the redemption rate can be calculated and depend on the face value of the coupon in our model.

A conventional monopolist does not have the ability to acquire individual consumers’ preferences and differentiate prices for consumers according to their willingness to pay. For instance, a retailer cannot discriminate and ask a different price from each consumer who steps in to the store. First of all, this is not possible as the retailer has no information regarding the consumers’ desire and need, therefore value, for the product. Second of all, once the retailer’s practice is known, consumers might revolt and the whole idea of price discrimination may backfire. Finally, this practice may also be deemed illegal in certain situations if it hampers the competition (U.S. Code 2002).
As David Streitfeld puts it “few things stir up a consumer revolt quicker than the notion that someone else is getting a better deal.” (Streitfeld 2000, p. A01)

Given this fact, the use of coupons as a price discrimination tool is well justified. Consumers do not feel cheated; at least they do not revolt against it, if they discover that someone else bought the same product at a lower price using a coupon. This practice is also legal. Because, there is an effort, to a certain extent, required on the consumers side to use the coupon. Later in this section we will include this effect in our model. In the next section we will show what happens when this effect is removed altogether, namely when the disutility of using coupons is zero.

Despite the fact that targeting is possible to a certain degree even with conventional coupons, we do not address that case in this chapter. Shaffer and Zhang (1995) develop a model of coupon competition in a duopoly where consumer targeting based on panel data at household level is assumed possible.

4.2.1 Model

In our basic model, we assume that a conventional firm sends coupons with fixed face value to everyone but will restrict the number of available coupons. The firm’s coupons will reach only $\lambda$ fraction of consumers. Furthermore, we assume that each consumer is equally likely to receive a coupon from the conventional firm. Therefore, the probability that a consumer will get a coupon is $\lambda$. The segment of consumers who would not buy the product without a coupon will not buy if they do not receive a coupon. If they receive a coupon, their purchase decision will depend on the face value of the coupon. The marginal consumers’ decision will be based on whether or not they receive
a coupon. The following equality will hold for the marginal consumers who receive the coupon with face value $f$ and are indifferent between buying and not buying, see Figure 2.

$$r - p - dt - xt + f = 0 \quad (4.2)$$

Where $x$ denotes to the segment of consumers who buy the product only if they receive the coupon. Since $r - p - dt = 0$ from Equation (4.1), we have $x = \frac{f}{t}$. Of course this is the case if they receive a coupon. Therefore, the expected value of $x$, the location of the marginal consumers on the unit circle, can be found as $E(x) = \bar{x} = P(\text{Coupon}) x = \frac{\lambda f}{t}$.

This same result can be derived if we look at the problem from a different perspective. Let us assume that the marginal consumers get the coupon but the face value of the coupon is a random variable. The face value of the coupon is $f$ with probability $\lambda$ and it is 0 with probability $1-\lambda$. Therefore, the expected value of the coupon face value is $E(f) = \bar{f} = \lambda f + (1-\lambda)0 = \lambda f$. Thus, the expected value of $x$ can be calculated as

$$\bar{x} = E(\lambda) = E\left(\frac{f}{t}\right) = \frac{E(f)}{t} = \frac{\lambda f}{t}.$$

Next, we model the cost of distributing coupons. For traditional paper coupons, the marginal cost of distributing coupons is assumed to be increasing in the number of consumers that receive the coupons (Bester and Petrakis 1996). Bester and Petrakis state that this is a standard assumption in advertising literature (see Butters 1977, Grossman and Shapiro 1984) and explain this as follows:

The underlying idea is that the manufacturer can distribute coupons via mail by placing ads in a set of magazines or newspapers. The probability that a given consumer receives a coupon through one of these media is independent of receiving a coupon through the other media. In this case, the cost of making sure that a fraction $\lambda$ of consumers receives at least one
coupon becomes a convex function of $\lambda$. (Bester and Petrakis 1996, p. 232)

As in Bester and Petrakis (1996), our cost function $C(\lambda)$ will take the following form: $C(\lambda) = a\lambda^\alpha \quad \alpha > 1$. We use $\alpha = 2$ in our analysis and this results in a quadratic cost function. The quadratic form is chosen because it complies with all the assumptions made in Bester and Petrakis (1996) and it is more tractable.

Additionally, this model considers the disutility of using coupons. As mentioned earlier, Narasimhan (1984) asserts coupons as price discrimination devices. He states that the reason for the use of coupons is to price discriminate between consumers with high demand elasticity and those with low demand elasticity. The consumers who enjoy the benefits of coupons also incur certain disutility. Narasimhan (1984, p. 131) lists the costs as “cost of organization” (includes “cost of organizer,” “time spent in periodically organizing coupons,” etc.), “cost of looking through magazines,” and “time spent in storing and retrieving coupons,” etc. All of these cost factors boil down to “disutility due to lost time” which varies tremendously from one person to the next. The consumers whose time is not as precious as others will tend to search for coupons more often than the consumers whose time is extremely valuable. By providing coupons, firms will lure consumers who would not buy the product without a coupon. With the availability of a coupon, these consumers will buy the product. Incidentally, firms will be able to charge higher prices to consumers who do not use coupons.

There is a disutility associated with using the coupon. A consumer’s disutility is represented by $\kappa_i$, which defined as consumer $i$’s disutility of using the coupon. We assume that consumers who are closer to their ideal product will have less incentive to use coupons. Therefore the disutility of using coupons, in the context of the unit circle
model, will decrease as we move away from the point where the product is marketed. The decrease per unit distance of circle is designated by \( \alpha \). Additionally, \( Z \) represents the maximum disutility of using a coupon. Since the maximum distance is \( \frac{1}{2} \) the minimum disutility of using coupons for any consumer will be \( Z - \frac{\alpha}{2} \). Hence, the disutility of using coupons is described by \( \kappa = Z - \alpha d \). We do not restrict this value to be strictly positive.

If it becomes negative for some value of \( d \), it is treated as an additional incentive to use the firm’s coupon. As a matter of fact what \( \alpha \) represents is a measure of difference between price sensitivity of coupon users and nonusers. In a way, we model this price sensitivity difference by reducing the fit cost \( (t) \) for coupon users by \( \alpha \). Shaffer and Zhang (1995) use a similar adjustment and assume a lower fit cost for coupon users compared to nonusers. Figure 2 visualizes the dynamics of this model.

\[ K = Z - \frac{\alpha}{2} \]

Figure 2 Consumers’ Disutility of Using Coupons

Because of the disutility of using coupons some consumers will not use the firm’s coupon even if they receive it. This is true especially in the segment of consumers who would buy at the current price without a coupon; therefore, in this segment, some
consumers will buy using the coupon and some consumers will buy without using the coupon. Since the disutility of using coupons decreases as consumers are away from their preferred product, we will have new marginal consumers in this segment. They are located at points $F$ and $G$, at distance $y$ from where the product is marketed. Therefore, their disutility of using the coupon will be $\kappa = Z - \alpha y$ (see Figure 3).

![Figure 3 Market Structure With Disutility of Using Coupons](image)

These new marginal consumers will use the coupon if the additional benefit from using the coupon, i.e., the face value of the coupon ($f$), outweighs the disutility of using it ($\kappa$). They will be indifferent between using and not using if the face value of the coupon equals the disutility. The following holds for these marginal consumers located at points $F$ and $G$.

\[
f - \kappa_m = 0,
\]
\[
f - (Z - \alpha y) = 0,
\]
\[
y = \frac{Z - f}{\alpha}.
\]

Consumers whose distance to the product is larger than $y$ will use the coupon if they receive one, because their disutility of using the coupon is lower than that of the
marginal consumers. This means that the size of the consumer segment that will use the coupon and would have bought the product without the coupon anyway, had they not received it, is 

\[
2(d - y) = 2 \left[ d - \frac{Z - f}{\alpha} \right] = 2 \left[ \frac{r - p}{t} - \frac{Z - f}{\alpha} \right].
\]

We can think of this segment as the firm’s loss due to couponing. The firm sacrifices the value \( f \) for each consumer in this segment in order to extend its reach. As for the consumers who will only buy the product if they receive a coupon, the situation is different. Because for these consumers the net utility without the coupon is negative, therefore the face value of the coupon should exceed not only the disutility of using the coupon but also the excess fit cost these consumers incur. However, the disutility of using coupons is lower for this segment of consumers, as they are away from their ideal product. As in the previous model without coupons we will assume that the marginal consumers are located at distance \( d + x \) away from the point where the product is offered. The points \( D \) and \( E \) in Figure 3 show the location of these marginal consumers. We make an additional assumption that the unit fit cost incurred is greater than the unit decrease in the disutility of using coupons, \( t > \alpha \).

For the marginal consumers who are indifferent between buying and not buying, the following equation holds.

\[
r - p - dt - xt + f - [Z - \alpha(d + x)] = 0
\]

(4.3)

After some algebra we have \( x = \frac{f - Z + \alpha d}{t - \alpha} \), i.e. \( x = \frac{f - Z + \alpha \left( \frac{r - p}{t} \right)}{t - \alpha} \). Of course, this is the case if the marginal consumers receive a coupon. Therefore, the expected value of \( x \), the location of the marginal consumers on the unit circle, can be found as
Given the expected value of the distance $x$, we can calculate the profits of the conventional firm in the existence of disutility for using coupons.

$$\Pi_{Total} = \Pi_{NC} + \Pi_c - C(\lambda)$$

We have the following two profit formulae for the no-coupon and coupon segments of the market.

$$\Pi_{NC} = 2[pd - \lambda f(d - y)] = 2\left[p\left(\frac{r-p}{t}\right) - \lambda f\left(\frac{r-p}{t} - \frac{Z-f}{\alpha}\right)\right]$$

$$\Pi_c = 2\bar{x}(p-f) = 2\lambda \left[\frac{f-Z+\alpha\left(\frac{r-p}{t}\right)}{t-\alpha}\right](p-f)$$

In the no-coupon segment, the consumers who get the coupon, $\lambda$ fraction, use it and pay $(p-f)$ whereas the others, $(1-\lambda)$ fraction, pay the full price $p$. As mentioned earlier the cost of distributing coupons is a quadratic function of $\lambda$. Hence the profit of the conventional firm in the existence of disutility for using coupons is

$$\Pi_{Total} = 2\left[p\left(\frac{r-p}{t}\right) - \lambda f\left(\frac{r-p}{t} - \frac{Z-f}{\alpha}\right)\right] + \lambda \left[\frac{f-Z+\alpha\left(\frac{r-p}{t}\right)}{t-\alpha}\right](p-f) - a\lambda^2$$

$$\Pi_{Total} = 2\left[\left(\frac{rp-p^2}{t}\right) - \frac{\lambda fp + \lambda fZ - \lambda f^2}{t}\right] - a\lambda^2$$

$$+ \frac{\lambda}{t-\alpha} \left[pf - f^2 - Zp + Zf + \frac{arp}{t} - \frac{arf}{t} - \frac{\alpha p^2}{t} + \alpha pf\right] - a\lambda^2$$
Therefore the conventional firm tries to solve the following problem with disutility of using coupons:

\[
\begin{align*}
\text{Max}_{p,f,\lambda}\left[\Pi_{\text{total}}\right] &= 2\left[\left(\frac{rp - p^2}{t}\right) - \frac{\lambda fp}{t} + \frac{\lambda fZ}{\alpha} - \frac{\lambda f^2}{\alpha}\right] \\
&\quad + \frac{\lambda}{t - \alpha}\left[pf - f^2 - Zp + Zf + \frac{arp}{t} - \frac{arf}{t} - \frac{\alpha p^2}{t} + \frac{\alpha pf}{t}\right] \\
&- a\lambda^2
\end{align*}
\]

(4.4)

Subject To

\[
\begin{align*}
0 \leq \lambda &\leq 1 \\
0 \leq p &\leq r \\
0 \leq f &\leq p \\
0 \leq \frac{r - p}{t} &\leq \frac{1}{2} \\
0 \leq \frac{f}{t} &\leq \frac{1}{2} - \frac{r - p}{t} \\
0 \leq \frac{Z - f}{\alpha} &\leq \frac{r - p}{t}
\end{align*}
\]

(4.5) (4.6) (4.7) (4.8) (4.9) (4.10)

The constraints given in equations (4.5)-(4.7) guarantee nonnegative and feasible values for the decision variables \(\lambda, p, f\). Equation (4.8) makes sure that the size of the no coupon segment \((d)\) is between 0 and \(\frac{1}{2}\). Equation (4.9) is used for making sure that the size of the coupon segment is nonnegative and not greater than the leftover market size \(\frac{1}{2} - d\). Finally, Equation (4.10) guarantees that \(y\) is nonnegative and less than or equal to \(d\). The solution to this problem is given in the Appendix A and we have \(p^* = \frac{r}{2}, f^* = \frac{Z}{2}\)
and $\lambda^* = \frac{(ar-Zt)^2}{4aat(t-\alpha)}$. We additionally have $d^* = \frac{r}{2t}$, $x = \frac{ar-Zt}{2t(t-\alpha)}$, and hence the market coverage is $k^* = \frac{r}{t} + \frac{(ar-Zt)^3}{4aat^2(t-\alpha)^2}$. We can also calculate a very critical performance measure for couponing at this point, the Coupon Redemption Rate (CRR), as follows. As First, we need to find the total fraction of consumers who use coupons, denoted “Total Coupon Users (TCU),” as $TCU = \frac{(ar-Zt)^3}{4a\alpha^2t(t-\alpha)^2}$. Then, we find the CRR as $CRR = \frac{TCU}{\lambda} = \frac{ar-Zt}{\alpha(t-\alpha)}$. Finally, the total profits of the firm can be calculated by substituting $p^*$, $f^*$ and $\lambda^*$ in the profit formula as $\Pi^*_{\text{Total}} = \frac{r^2}{2t} + \frac{(ar-Zt)^4}{16a\alpha^2t^2(t-\alpha)^2}$.

4.2.2 Discussion and Analysis of Results

By using a conventional coupon, a monopolist achieves strictly higher profits. However, the firm does not change its price structure compared to the no coupon case. The same price $\frac{r}{2}$ still prevails as the optimal price even with the use of coupons. This is contrary to the conventional wisdom that the firm will increase its regular price in the presence of coupons. The model allows the firm to price discriminate between coupon users and nonusers. While no coupon users are charged $\frac{r}{2}$, coupon users are charged $\frac{r-Z}{2}$. The face value of the coupon is given by $\frac{Z}{2}$ and it depends only on the maximum disutility of using coupon ($Z$). No other parameter of the model affects the coupon face value.
Another important result is that the cost of coupon distribution does not affect the existence of couponing. It does, however, effect the fraction of consumers who get a coupon and the effectiveness of couponing in terms of additional profit it generates.

As we mentioned earlier, the profit of the firm increases with coupons and the new profit value is $\frac{r^2}{2t} + \frac{(ar-Zt)^4}{16a^2 \alpha^2 t^2 (t-\alpha)^2}$. The first part of this value, $\frac{r^2}{2t}$, is exactly equal to the profit in the no coupon case. The second part represents the net benefit of the couponing. As stated earlier, there are some consumers who will use the coupons if they receive one; at the same time these same consumers would be willing to buy the product even without the coupon. Therefore, the total value of coupons redeemed by these consumers can be considered as a loss due to couponing. We call this value, $\pi_l$, the profit loss. Similarly, we have consumers who will buy the product only with the provision of a coupon. The purchases of these consumers are the gain from couponing. We call this value, $\pi_g$, profit gain. The trade-off between these two monetary figures decides the use of coupons in the market. If the loss due to couponing outweighs the benefits, no coupons are provided; otherwise coupons are provided proportional to the net benefit amount. We can summarize results presented here in the following proposition.

**PROPOSITION 1:** Only if $r>Z$, $t>\alpha$ and $\frac{r}{t} \geq \frac{Z}{\alpha}$, will a firm introduce coupons with face value $\frac{Z}{2}$. When this occurs it will distribute coupons to $\frac{(ar-Zt)^3}{4a \alpha t^2 (t-\alpha)}$ percent of the consumers, increasing the total profit by $\frac{(ar-Zt)^4}{16a^2 \alpha^2 t^2 (t-\alpha)^2}$. Otherwise, it is not profitable for the firm to introduce coupons.
COROLLARY 1: If the firm introduces coupons the redemption rate of coupons will be \( \frac{ar - Zt}{\alpha (t - \alpha)} \).

In this proposition, the condition \( t > \alpha \) has a special meaning. Parameter \( t \) is the reduction in consumers’ utility as consumers fall away from the current product available in the market. Similarly \( \alpha \) is the reduction in consumers’ disutility of using the coupon per unit distance. As consumers’ tastes fall away from the available product, reduction in their willingness to pay should be more than the reduction in disutility of using the coupon so that the firm can price discriminate. If this condition fails, the price discrimination device will not work since no coupon face value will be able to separate two consumers. Suppose this condition does not hold and we have two consumers, 1 and 2. The consumers’ distances from the product on the unit circle are \( d_1 \) and \( d_2 \) respectively, where \( d_1 \neq d_2 \). If \( d_1 > d_2 \) for any coupon face value and when consumer 2 uses the coupon consumer 1 will also use it. The exact opposite is true if we have \( d_1 < d_2 \).

4.2.3 Numerical Examples

In this section we will give a set of numerical examples to help explain what happens when we have a disutility of using the coupon for consumers. The results of examples 1-6 are given in Table 4. In the first example we assign values to our parameters as given in the second column of Table 3. Specifically we let \( r = 3, t = 5, \alpha = 2, Z = 1 \) and \( a = 0.05 \). The values of decision variables and other results are given in the table. The values of parameters for example 1 are totally arbitrary and chosen to illustrate the workings of our model. The constraints given in Table 3 are all satisfied for
these parameter values. The results suggest that it is optimal for the monopolist to set the price at $1.5. This price value would cover the 60% of the market without coupons. To extend its market reach and improve its profits the firm introduces a coupon with face value of $.50 and distributes this coupon to consumers and the coupon reaches an effective 16.67 percent of the consumer population. 2.78 percent of consumer population or 16.67 percent of consumers who receive the coupon actually redeem the coupon.

In examples 2-5 we keep all but one of the parameters at their initial values and find the feasible range for the free variable that satisfies all the constraints. Then we find the values of all decision variables and results in terms of this free variable. Then we plot the optimal profit ($\Pi^*$) and the optimal coupon distribution intensity ($\lambda^*$) against each parameter ceteris paribus. The resulting graphs are given in Figure 4 Figure 8. Figure 4 shows the effect of the common reservation price $r$ on a firm’s profit and coupon distribution intensity. As the common reservation price increases, both the profit and the coupon distribution intensity increase. Figure 5 illustrates the effect of the coupon distribution cost, $a$. The coupon distribution cost, as mentioned earlier, does not determine the existence of coupons. It does however affect the amount of coupons distributed dramatically. If the coupon distribution cost decreases, profit and coupon distribution intensity sharply decline. In Figure 6, we analyze the effect of the decrease in the disutility of using coupons parameter, $\alpha$. Both the profit and the coupon distribution intensity increase with increased $\alpha$. However its effect on profit is less than the effect of the common reservation price. Figure 7 shows the effect of the fit cost, $t$. The fit cost has an effect that is very similar to the effect of the common reservation
price. Finally, Figure 8 looks at the combined effect of the fit cost and the decrease in the disutility of using coupons.

Table 3 Assumptions of the Model

<table>
<thead>
<tr>
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<th>Assumption</th>
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<tbody>
<tr>
<td>1</td>
<td>$0 \leq d \leq \frac{1}{2} \Rightarrow 0 \leq \frac{r - p}{t} \leq \frac{1}{2} \Rightarrow r \geq p$ and $r \leq p + \frac{t}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$t - \alpha \geq 0 \Rightarrow t \geq \alpha$</td>
</tr>
<tr>
<td>3</td>
<td>$0 \leq \lambda \leq 1 \Rightarrow 0 \leq \frac{(ar - Zt)^2}{4aat(t - \alpha)} \leq 1$</td>
</tr>
<tr>
<td>4</td>
<td>$x \geq 0 = \frac{ar - Zt}{2(t - \alpha)} \geq 0 \Rightarrow ar - Zt \geq 0 \Rightarrow ar \geq Zt$</td>
</tr>
<tr>
<td>5</td>
<td>$0 \leq k \leq 1 \Rightarrow 0 \leq \frac{r}{t} + \frac{(ar - Zt)^2}{4aat^2(t - \alpha)^2} \leq 1$</td>
</tr>
<tr>
<td>Parameters</td>
<td>Example 1</td>
</tr>
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<td>-----------</td>
</tr>
<tr>
<td>$r$</td>
<td>3</td>
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<td>$t$</td>
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<tr>
<td>$Z$</td>
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</tr>
<tr>
<td>$a$</td>
<td>0.05</td>
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<table>
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<tr>
<th>Decision Variables</th>
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<tr>
<td>$p$</td>
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<td>$\pi$</td>
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<tr>
<td>Parameters</td>
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<td>TCU</td>
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<td>CRR</td>
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<tr>
<td>Constraints</td>
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</tbody>
</table>
Figure 4 Profit and Coupon Distribution Intensity vs. r
Figure 5 Profit and Coupon Distribution Intensity vs. a
Figure 6 Profit and Coupon Distribution Intensity vs. $\alpha$
Figure 7 Profit and Coupon Distribution Intensity vs. $t$
Figure 8 Profit and Coupon Distribution Intensity vs. t and α
CHAPTER 5
SPATIAL MODELS OF INTERNET COUPONS

Internet coupons are revolutionary mechanisms to achieve one-to-one marketing capabilities on the Internet. Developments in the field of customer profiling enable firms to target consumers much more precisely than it was possible ever before. Customer profiling is a part of Customer Relation Management (CRM), a rapidly growing area of electronic commerce. CRM is a huge market; consulting firm IDC estimates CRM revenues to increase from $61 billion in 2001 to $148 billion in 2005 (NYU Stern 2001).

The Internet heralds fundamental changes in all functions of an organization, particularly marketing. Technological improvements in providing rich-media content with low-bandwidth requirements, such as Macromedia Inc.’s Flash, combined with increased accessibility of broadband networking will help fuel the Internet advertising boom. While Internet advertising is increasing its share, it becomes more and more difficult to attract consumers and make them respond to online offerings. The response rate (also called the click through rate) to online banner ads dropped drastically from 25-30% to less than 1% in recent years (Morgan Stanley Dean Witter 2000). To solve this attention deficit problem, firms seek solution by tying online promotions, such as rewards, rebates and coupons, with online advertising campaigns. Out of all promotion devices, coupons are the most important and effective in a firm’s sales promotion plan (Nevo and Wolfram 2001). As we mentioned in Chapters 1 and 2 there are many forms of coupon provisions in use today including Internet coupons and there are a variety of different ways to deliver coupons to consumers over the Internet. With the technology
available today, it is possible to target individual consumers using Internet coupons. In this chapter we model a monopolist’s decision to use an Internet coupon given that she can effectively target consumers according to their tastes for the product to improve her profits. This chapter develops models to compare different Internet couponing alternatives. We develop an analytical framework to investigate how Internet coupons can be used to target consumers and efficiently price discriminate to improve a firm’s profit by extending Salop’s spatial model (Salop, 1979) to address issues related to Internet coupons.

As stated earlier price discriminating between consumers according their price sensitivity and charging more to less price sensitive consumers and vice versa is one of the aims of couponing. Narasimhan (1984) showed evidence regarding the price discrimination benefits of coupons. We first show in this chapter that treating Internet coupons the same way as conventional paper coupons will have some unintended consequences. One of the reasons why Internet coupons provide such a high redemption rate is their ease of use. High redemption rates may be good news to coupon providers given that only targeted consumers see their Internet coupons. If their targeted coupons become mass-media coupons, through unlawful duplication or incorrect distribution strategy by providers, the results may or may not be what they were intended to be. We show that if this occurs, price discrimination will not be possible. If the use of Internet coupons becomes extremely easy such that everyone who receives a firm’s coupon redeems it, our model indicates that the firm’s optimal choice is not to provide coupons at all. Any level of coupon distribution hurts the firm’s profitability. To benefit from this new form of providing coupons over the Internet, firms need to understand its advantages
and disadvantages. If a firm distributes coupons randomly over the Internet, then it should make sure consumers incur some disutility when they use these Internet coupons. The firm should create some costs such as requiring registration with the site, downloading software, printing a paper coupon and so on\(^1\). By doing so, the firm will benefit from the low cost of distribution on the Internet as well as price discrimination.

Next, we present the model of a firm that provides a fixed value coupon over the Internet but it targets the coupon to certain consumers. If a firm chooses this alternative over the mass distribution of its coupons, it should make every effort to decrease the discomfort consumers incur using the coupon so that every one in the targeted segment uses the coupon\(^2\).

Finally, we introduce the model of the changing face value Internet coupons. Using fixed face value Internet coupons, firms benefit from the low cost of distributing coupons over the Internet. However, this approach does not extract the benefits of one-to-one marketing ability afforded by the Internet. With the advent of tracking and personalization technologies, firms can individually target each consumer and tailor its coupon according to individual consumer’s preferences and willingness to pay. In this chapter, we explore the models of changing face value Internet coupons involving two

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\(^1\) This is exactly what is happening with some of the Internet coupon providing firms. For instance, CoolSavings\(^\circ\).com and Coupons Inc. require a software download. ValuPage\(^\circ\) grants no immediate cash discount, the reward can only be redeemed at the next visit.

\(^2\) Planet U\(^\text{TM}\) attempts to provide paperless coupons that can be directly tied to a consumer’s frequent shopper card and used towards an on- or off-line purchase. The company also provides tools to target coupons to consumers. The targeted coupons distributed this way will cause very little discomfort to consumers.
cases where the firm has perfect information or imperfect information about the consumers. In the next section we look at the mass distribution of Internet coupons.

5.1 Mass Distribution of Coupons over the Internet with no Targeting

When a firm does not have the ability to acquire individual consumers’ preferences, it cannot differentiate its coupon provision among consumers according to consumers’ willingness to pay, or some proxy thereof. Its only choice is to distribute coupons randomly, through traditional coupons distribution methods or over the Internet. As the cost of distribution over the Internet is lower than the traditional distribution channels, the firm will choose to distribute over the Internet. With a traditional paper coupon there is an effort, to a certain extent, required on the consumers’ side to use the coupon. This is what allows firms to use coupons as price discrimination tools. In Chapter 4 we modeled this effort and find that this effort is very critical to the success of a couponing campaign. For some forms of Internet couponing that we describe earlier, this consumer’s effort on using the coupons distributed over the Internet is negligible. And those are the forms of Internet coupons we consider in this chapter. In this section we show the consequence to the firm when the disutility of using Internet coupons becomes zero.

Assume that the firm will give coupons to $\lambda$ fraction of consumers randomly. Furthermore, we assume that each consumer is equally likely to receive a coupon from the firm. Therefore, the probability that a consumer will get a coupon is $\lambda$. The costs of distributing coupons in this case will be linear in this fraction (or probability). This is because the marginal cost of distributing coupons over the Internet is constant in the
number of consumers that receive the coupons. Our cost function $C(\lambda)$ will take the following form:

$$C(\lambda) = a_\ell + b_\ell \lambda.$$  

In this case, consumers who would buy without a coupon will still buy using the coupon, if they receive one, since there is no disutility associated with using the coupon. Hence, $\lambda$ proportion of the segment $d$ will use the coupon. The segment of consumers who would not buy the product without a coupon will not buy if they do not receive a coupon. If they receive a coupon, their purchase decision will depend on the face value of the coupon. The marginal consumers’ decision will be based on whether or not they receive a coupon. Equation (5.1) will hold for the “marginal” consumers who receive the coupon with the face value $f$ and are indifferent between buying and not buying, see Figure 9. Here $x$ denotes the size of the segment of consumers who buy the product only if they receive the coupon.

$$r - p - dt - xt + f = 0$$  

(5.1)

Since $r - p - dt = 0$ from Equation 4.1, we have $x = \frac{f}{t}$. This is the case if they receive a coupon.

Therefore, the expected value of $x$, the location of the marginal consumers on the unit circle, can be found as $E(x) = \bar{x} = \Pr\{\text{Receiving a coupon}\} \cdot x = \frac{\lambda f}{t}$.

This same result can be derived if we look at the problem from a different perspective. Assume that the marginal consumers get the coupon for sure but the face value of the coupon is a random variable. The face value of the coupon is $f$ with probability $\lambda$ and it is 0 with probability $1 - \lambda$. Therefore, the expected value of the
The coupon face value is \( E(f) = \bar{f} = \lambda f + (1 - \lambda)0 = \lambda f \). Hence the expected value of \( x \) can be calculated as follows.

\[
\overline{x} = E(x) = E\left(\frac{f}{t}\right) = \frac{E(f)}{t} = \frac{\bar{f}}{t} = \frac{\lambda f}{t}.
\]

Given the expected value of the distance \( x \), we can calculate the profits of the firm.

\[
\Pi_{Total} = \Pi_{NC} + \Pi_{C} - C(\lambda)
\]

where the profit formulae for the no coupon and coupon segments of the market are

\[
\Pi_{NC} = 2[1 - \lambda]dp + \lambda d(p - f) \quad \text{and} \quad \Pi_{C} = 2\bar{x}(p - f) = 2\lambda x(p - f).
\]

In the \( d \) segment the \( \lambda \) fraction of consumers who get the coupon will use it and pay \((p - f)\) whereas the others \((1 - \lambda)\) fraction pay the full price \( p \). As mentioned earlier the cost of distributing coupons is assumed to follow a linear function of \( \lambda \), \( C(\lambda) \). Hence the profit of the firm is

\[
\Pi_{Total} = 2[(1 - \lambda)dp + \lambda d(p - f)] + 2\lambda x(p - f) - a_i - b_i \lambda.
\]

By substituting Equation 4.1 into the objective function and after some algebra, we have the following profit function for the monopolist.

\[
\Pi_{Total} = \frac{2}{t}\left[tp - p^2 - \lambda rf + 2\lambda pf - \lambda f^2\right] - a_i - b_i \lambda.
\]
Then the firm solves the following problem.

\[
\begin{align*}
\text{Max}_{p,r,\lambda} & \left[ \frac{2}{t} \left[ rp - p^2 - \lambda rf + 2\lambda pf - \lambda f^2 \right] - a_i - b_i \lambda \right] \\
\text{Subject To} & \begin{align*}
0 & \leq \lambda \leq 1 \\
0 & \leq p \leq r \\
0 & \leq f \leq p \\
0 & \leq \frac{r-p}{t} \leq \frac{1}{2} \\
0 & \leq \frac{f}{t} \leq \frac{1}{2} - \frac{r-p}{t} 
\end{align*}
\end{align*}
\] (5.2)

The constraints given in equations (5.3), (5.4), and (5.5) guarantee nonnegative and feasible values for the decision variables \(\lambda, p, f\). Equation (5.6) makes sure that the size of the no coupon segment \(d\) is between 0 and \(\frac{1}{2}\). Finally, Equation (5.7) ensures that the size of the coupon segment is nonnegative and not greater than the leftover market size \(\frac{1}{2} - d\).

The solution to this model as well as the solutions to subsequent models in this chapter has been delegated to the Appendix A. The solution of this model leads to

\[ (1 - \lambda^*) f^* = 0 \]

This implies two cases – either \(f^*\) is 0 and \(\lambda^*\) is free or \(\lambda^*\) is 1 and \(f^*\) is free. In the first case, the model suggests no issuance of a coupon, as zero face value implies no coupon distribution. In the second case, where \(\lambda^* = 1\), we have \(\Pi^*_{\text{final}} = \frac{r^2}{2t} - a_i - b_i\). This profit figure is less than the figure for the no coupon case. Therefore, the firm’s optimal choice
will be to provide no coupons at all. The mass distribution of coupons over the Internet does not help the profitability of the firm if the coupons are distributed over the Internet without targeting consumers. This results from the fact that the firm will not be able price discriminate when everyone uses the coupons.

In order for the firm to still be able to price discriminate, it needs a mechanism to impose certain effort for the consumers to use coupons, even with Internet coupons. In Chapter 4 we examine the case for mass media distributed paper coupons that impose disutility of usage to consumers and find similar conclusions, the only difference being the increased profit in this case due to lower costs of coupon distribution over the Internet. To use the Internet for mass distribution of coupons without targeting does not allow firms to realize all the benefits the Internet has to offer. In the next section, we analyze the targeted distribution of Internet coupons as an alternative to mass distribution.

5.2 Spatial Models of Internet Targeted Coupons

Internet coupons are revolutionary mechanisms to achieve one-to-one marketing capabilities for the firm as the Internet technology enables the firm to target individual consumers. We model a monopolist’s decision to use an Internet coupon given that it can effectively target consumers according to their tastes for the product to improve its profits. In Section 5.2.1, we first analyze a hypothetical case where a firm can target consumers perfectly without incurring any costs. This model is rather unrealistic and is used only as the base model for comparison with more realistic models that include costs of customer profiling and imperfect targeting introduced in Section 5.2.2. With imperfect targeting and costs of customer profiling, the firm has the alternative to provide
consumers with a coupon with fixed face value or a different coupon with changing face value for each individual consumer to induce them to buy the product.

We follow a two-period pricing/couponing process. In both models in Section 5.2.2 we assume that the consumers who would buy the product without a coupon reveal themselves in the first period, and the firm introduces coupons to the rest of the consumers in the second period. The firm targets those consumers with a high fit cost who had not been able to buy the product at the regular price by introducing coupons in the second period.

5.2.1 Targeted Coupons with Free Perfect Information

With the flexibility of the Internet, firms can change the face value of their coupon offerings according to individual consumers’ preferences and shopping behavior. When the firm has detailed information regarding consumers’ shopping behavior and preferences, it can also estimate their willingness to pay to some extent. Instead of one same coupon, separate coupon for each consumer can be issued for immediate online use. Online consumer behavior can be analyzed in order to determine consumers’ desire for a product. According to the information gathered this way the firm can then decide on the coupon and the terms of the coupon. Our study focuses on the face value of the coupon. Other features such as the terms and the timing of the coupon are beyond the scope of this chapter. We assume that the coupon is given for one product and it is instantaneous as soon as the face value is determined. Coupons distributed this way will not impose any disutility of using.

With the help of coupons, a monopolist can charge higher prices to some consumers and lower prices to others and segment consumers into many segments. However, with traditional paper coupons the firm is limited to very small number of
different coupons. In practice, a conventional firm generally uses one coupon with the same face value for all consumers. Online marketing allows firms to target their consumers in ways that were not feasible before. Therefore, the firm can effectively target those consumers who should be given a coupon.

We again consider a monopolistic market for a single product with the exact same characteristics described in Chapter 4 – zero marginal production cost, product characteristics approach to product differentiation, unit circle model with the total number of consumers normalized to one, the common reservation price \( r \), and heterogeneous consumer preferences represented by the fit cost \( t \).

In the model of targeted coupons with perfect information, we assume that each consumer’s true location on the unit circle can be identified perfectly with an information gathering mechanism. We assume that the cost of the perfect information is constant and set to zero. This unrealistic assumption will be relaxed in subsequent models.

Figure 10 describes the essence of the model of targeted coupons with perfect information. The firm’s product is marketed at point \( B \). Assume that the two marginal consumers located at points \( A \) and \( C \) are indifferent between buying and not buying the product at price \( p \). Both \( A \) and \( C \) are distance \( d \) away from the product and hence incur fit cost \( dt \). For these consumers Equation 4.1 holds and we have \( d = \frac{r - p}{t} \). The profit for no coupon (NC) segment is given as earlier by \( \Pi_{NC} = 2\left(\frac{r - p}{t}\right)p \).

To entice those consumers who did not buy the product at price \( p \) in the first period, the firm decides to issue coupons. Instead of distributing one fixed face value Internet coupon, the firm can tailor the coupon for each individual consumer. The
firm’s choice of the coupon face value for individual \( i \), denoted by \( f_i \), is just enough to induce consumer \( i \) to buy the product. Since \( r - p - dt = 0 \), \( r - p - dt < 0 \) for all \( d_i > d \).

If we let \( x_i \) denote consumer \( i \)'s distance from the closest marginal consumer, we have \( d_i = d + x_i \). Thus, consumer \( i \)'s net utility for the product is

\[
- x_i t + f_i \geq 0.
\]

As we assume that the consumers who are indifferent will buy the product, the firm should set the face value of the coupon for consumer \( i \) equal to the incremental fit cost this consumer incurs compared to the marginal consumer. That is, at optimality we have \( f_i = x_i t \). Since we assume no cost of producing an extra unit of the product, the firm is willing to provide coupon face value up to \( p \), i.e., \( f_i \leq p \). Let \( m \) denote the consumer with the largest negative net utility for the product before receiving a coupon, and let \( x_m \) denote her distance from the marginal consumer. This implies \( f_m = p \) or similarly \( x_m t = p \). Hence, \( x_m = \frac{p}{t} \). Therefore, the firm will provide coupon face value,
\[ f_i = x_i t \text{ for all } x_i \leq \frac{P}{t}. \] The firm’s profit from the coupon segment can be calculated accordingly.

In this initial model we assume that the process of learning consumers’ location can be done without any costs. The firm’s profit from the coupon segment is

\[ \Pi_c = 2 \int_0^{x_m} (p - f)dx = 2 \int_0^{r} (p - xt)dx. \]

Hence, the total profit is given by

\[ \Pi_{Total} = \Pi_{NC} + \Pi_c = 2\left(\frac{r-p}{t}\right)p + 2 \int_0^{r} (p - xt)dx = \frac{2rp - p^2}{t}. \]

Then, the firm’s problem can be formulated as follows.

\[
\text{Max} \left[ \frac{2rp - p^2}{t} \right] \tag{5.9} \\
\text{Subject To} \]

\[ 0 \leq p \leq r \tag{5.10} \]

\[ 0 \leq \frac{r-p}{t} \leq \frac{1}{2} \tag{5.11} \]

\[ 0 \leq \frac{p}{t} \leq \frac{1}{2} - \frac{r-p}{t} \tag{5.12} \]

The constraint in Equation (5.10) is straightforward. Equations (5.11) and (5.12) are the same as Equations (5.6) and (5.7) presented earlier. We find the solution to this problem as (see Appendix A) \( p^* = r \), \( \Pi_{Total}^* = \frac{r^2}{t} \), and \( d^* = 0 \). The market coverage

\[ k^* = \frac{2r}{t} \]. When \( r \geq \frac{t}{2} \), it is optimal for the monopolist to serve all consumers. If \( r < \frac{t}{2} \), some consumers are not served at the monopolist’s optimal price.
The results of this basic model amount to perfect (first-degree) price discrimination. Each individual consumer is charged her true reservation utility for the product. However, this is not achieved through dynamic pricing but through the use of the changing face value coupons. The end results in these two alternatives are virtually the same; however, they generate different perceptions among consumers. When Amazon was found to use its enormous consumer information database to price discriminate, charging higher prices to loyal consumers that are frequent purchasers and supposedly less price sensitive, the consumers’ reaction was nothing but an adverse one (Streitfeld, 2000). The firm denied the allegations regarding dynamic pricing and claimed that it was merely running price tests. Even though dynamic pricing has been effectively used in other industries like airline and travel industries, its use by an online firm to sell DVDs stirred up a lot of emotion among consumers.

The situation is rather different with coupons. The use of coupons as price discrimination tool does not seem to have the same adverse effect that dynamic pricing has on consumers. Jupiter Media Metrix analyst Jared Blank support this viewpoint:

> Merchants that want to price a new product dynamically (especially one that consumers will discuss in chat rooms) should send coupons of differing values to customers, rather than adjusting the price of the item on the site. Consumers are not offended when others receive coupons of differing values; this happens often with consumer packaged goods. (Zaret 2001, p. 1).

One of the possible reasons for this difference between dynamic pricing and coupons may be the fact that the price is announced and known by all consumers when a coupon is used as a price discrimination tool. All consumers observe the same disclosed price but not necessarily the coupons when they visit the firm’s online establishment. With this use of coupons consumers might very well feel that they are getting a good deal
on the product. This might be the reason why price discriminatory couponing is not likely to receive adverse reaction from consumers. Much research is needed to find out the exact reasoning of consumers. However, it is beyond the scope of this study. What we are interested in is not why consumers react differently but, given these different reactions, what the firm should do to effectively price discriminate and serve more consumers, improving its profits along the way. In the next section, we relax some of the assumptions made in the basic model and analyze Internet coupon targeting with imperfect and costly information.

5.2.2 Targeted Coupons with Imperfect Information

With imperfect and costly information, the firm can choose between providing a fixed face value coupon or the changing face value coupons to different consumers. In the following two subsections, we present models for these two alternatives. We first analyze the fixed face value coupons case.

5.2.2.1 Fixed face value Internet coupons to targeted consumers

We model the couponing process as a two-period decision process. As the consumers who buy without the coupon reveal themselves in the first period, the firm has no interest in targeting these consumers. It will issue its Internet coupon in the second period. It will not give coupons to those consumers who would buy its product at price $p$ and let them buy the product without a coupon in the first period. Figure 11 displays the dynamics of this couponing model. The marginal consumers in the no coupon segment are located at points $A$ and $C$, and Equation 4.1 given in the previous chapter holds at these two points, i.e. $r - p - dt = 0$. In the segment where the coupon is provided, the marginal consumers who are indifferent between buying and not buying are located at points $D$ and $E$ (see Figure 11). Again, we model the case of Internet coupons, such as
Planet U™’s paperless coupons, where the disutility of collecting and using the coupon is negligible. Therefore, Equation (5.1) holds for the marginal coupon users at point $D$ and $E$, i.e., $r - p - dt - xt + f = 0$. Here $x$ is described as in Section 4, the segment of consumers who will buy the product only if they receive the coupon. As $r - p - dt = 0$ from Equation 4.1, we have $x = \frac{r - p - dt + f}{t} = \frac{f}{t}$.

In this section we maximize profit by taking both the price and the face value of the coupon as decision variables. We would like to see if the optimal price changes with the provision of coupons. Furthermore, the firm will want to have the flexibility to change its pricing structure anticipating that it will provide coupons later on. We assume that the customer profiling technology has constant returns to scale as Amazon and Yahoo can acquire consumers’ “purchasing and preference profile” information at a constant marginal cost for all consumers (Dewan et al. 2000). Therefore the firm
providing Internet coupons will incur the cost of customer profiling as a linear function of the coupon segment \(x\). The firm will have to observe consumers to decide whether or not the consumers would buy the product without coupons if it sets the face value at \(f\). We assume that the firm incurs customer profiling cost \(b_2\) per unit distance of observed market segment. We have \(\Pi_{\text{Total}} = \Pi_{\text{NC}} + \Pi_{C} - C(x)\) where \(\Pi_{\text{NC}} = 2dp\), \(\Pi_{C} = 2x(p - f)\), and \(C(x) = 2b_2x\).

After some algebra we have \(\Pi_{\text{Total}} = \frac{2}{t} \left[ rp - p^2 + pf - f^2 - b_2f \right]\). Therefore the firm’s profit maximization problem can be formulated.

\[
\begin{align*}
\text{Max}_{p,f} & \left[ \frac{2}{t} \left[ rp - p^2 + pf - f^2 - b_2f \right] \right] \\
\text{Subject To} \quad & 0 \leq p \leq r \\
& 0 \leq f \leq p \\
& 0 \leq \frac{r - p}{t} \leq \frac{1}{2} \\
& 0 \leq \frac{f}{t} \leq \frac{1}{2} - \frac{r - p}{t}
\end{align*}
\]

(5.13)

The constrains given in Equations (5.14)-(5.17) and their interpretations are the same as those in Equations (5.4)-(5.7). The solution to this problem is characterized by (see Appendix A) \(f^* = \frac{r - 2b_2}{3}, \quad p^* = \frac{2r - b_2}{3}\) and \(x^* = \frac{r - 2b_2}{3t}\). Additionally,

\[
d^* = \frac{r + b_2}{3t}.
\]

Hence the market coverage is \(k^* = \frac{2}{3t} (2r - b_2)\). And finally the total profit is \(\Pi^*_{\text{Total}} = \frac{2}{3t} \left[ r^2 - rb_2 + b_2^2 \right]\).
The following proposition summarizes the results of this model.

**PROPOSITION 2:** Only if \( r > 2b_2 \) and \( r < \frac{3t - b_2}{2} \), will a firm introduce fixed face value Internet coupons. When this is the case, the face value of the coupon is \( \frac{r - 2b_2}{3} \) and the firm will distribute coupons to \( \frac{2(r - 2b_2)}{3t} \) percent of the consumers, increasing the total profit by \( \frac{(r - 2b_2)^2}{6t} \). Otherwise, it is not profitable for the firm to introduce coupons.

In this model it is optimal for the firm to increase its price for the no coupon segment and provide coupons to the rest of the consumers. Chapter 4 showed that the conventional paper coupon provider did not have to increase its price to maximize its profits with paper coupon promotions. It was optimal for the conventional coupon provider to charge the same price with or without coupons. However, with Internet coupons, firms should increase the price for consumers with high valuation for the product, i.e. the consumers that are not given a coupon will need to bear higher prices. Not only is this price higher than what the consumers given the coupon pay but also higher than the price that would be charged if no coupons were provided at all. Further analysis of this model is given in Section 5.3. In the next section, we introduce the changing face value coupons in the case of imperfect targetability.

### 5.2.2.2 Changing face value Internet coupons to targeted consumers

In this section we analyze the provision of the changing face value coupons when the firm’s targeting ability is less than perfect. Our aim is to find conditions under which the firm will choose the changing face value coupons over the fixed face value coupons.
in the presence of imperfect information. If the firm estimates consumer $i$’s location in terms of her distance to the closest marginal consumer to be $\hat{x}_i$ and the true location of the consumer $i$ turns out to be $x_i$, one of the three possible cases arises: i) if $x_i > \hat{x}_i$, i.e. the firm underestimates the consumer’s fit cost, the consumer will not buy as the coupon face value she is given is not enough to offset her fit cost; ii) if $x_i = \hat{x}_i$, i.e. the firm estimates the consumer’s fit cost perfectly, the consumer will be indifferent and we assume she will buy; iii) and finally, if $x_i < \hat{x}_i$, i.e. the firm overestimates the consumer’s fit cost, the consumer will buy however she will be given a coupon with a face value that is larger than the face value required to induce her into buying. Therefore, the firm will give extra discounts to consumer $i$.

Suppose that the firm’s customer profiling system works imperfectly. It underestimates or overestimates consumers’ fit cost from time to time. Assume that the system’s accuracy is independent of consumers’ preferences. That is, the system’s failure to identify a consumer’s fit cost is constant for all consumers and does not depend on the true state of the consumer’s fit cost. Suppose also that the system either overestimates or perfectly estimates a consumer’s fit cost with probability $q$, i.e. $q = \Pr\{x_i \leq \hat{x}_i\}$ for all consumers and $x_i \in [0, x_m]$. In a recent paper, Chen et al. (2001) use a similar kind of probability measure to model imperfect targetability in individual marketing setting. Given this probability, we can reformulate the firm’s maximization problem. Let $\pi_{ic}$ denote the firm’s profit from consumer $i$ in the coupon segment. The value $\pi_{ic}$, given the face value of the coupon, $f_i$, is given by
\[
\pi_{ic} = \begin{cases} 
p - f_i & \text{with probability } q \\
0 & \text{with probability } 1 - q \end{cases}
\]

Or, if we write the same value in terms of the consumer’s location on the unit circle,

\[
\pi_{ic} = \begin{cases} 
p - \hat{x}_i t & \text{with probability } q \\
0 & \text{with probability } 1 - q \end{cases}
\]

Thus, the expected profit from consumer \(i\) is \(E(\pi_{ic}) = q(p - \hat{x}_i t)\). The total profit of the firm from the entire coupon segment can be calculated by simply integrating its profit from individual consumers over the interval \(\hat{x}_i \in [0, x_m]\). In this case the firm may not be willing to provide coupons with a face value up to the price as there are costs associated with changing the face value of the coupon. The total profit within this range is

\[
\Pi_c = 2 \int_0^{x_m} q(p - \hat{x} t)d\hat{x}.
\]

The cost of customer profiling and providing the accuracy \(q\) is given as follows:

\[
C(x, q) = 2b_x x_m + a_x q^2.
\]

The firm incurs a linear customer profiling cost \(b_x\) similar to the fixed face value coupon case. We assume that the technology to estimate individual consumers’ location on the unit circle along with its accuracy level is exogenous to our model. This assumption reflects the fact that these technologies are not available in the entire spectrum of accuracy levels from 0 to 1. The firm’s choice of accuracy is not continuous. The firm solves its optimization problem for a set of given technologies with associated accuracy levels and picks the best one. We assume that the firm incurs a quadratic cost for achieving accuracy level \(q\), which represents the increasing marginal cost of accuracy provision. Hence, the firm’s total profit is given by
\[ \Pi_{\text{Total}} = \Pi_{\text{NC}} + \Pi_{\text{C}} - C(x,q) = 2(\frac{r-p}{t})p + 2 \int_0^{x_m} q(p-xt)dx - (2b_2x_m + a_2q^2). \]

After some algebra we have \[ \Pi_{\text{Total}} = \frac{2rp - 2p^2 + 2tpx_m - qx_m^2t^2 - 2tb_2x_m - ta_2q^2}{t}. \]

Therefore the firm solves the following problem.

\[ \underset{p,x_m}{\text{Max}} \left[ \frac{2rp - 2p^2 + 2tpx_m - qx_m^2t^2 - 2tb_2x_m - ta_2q^2}{t} \right] \]  
(5.18)

Subject To
\[ 0 \leq p \leq r \]  
(5.19)
\[ 0 \leq f \leq p \]  
(5.20)
\[ 0 \leq \frac{r-p}{t} \leq \frac{1}{2} \]  
(5.21)
\[ 0 \leq x_m \leq \frac{1}{2} - \frac{r-p}{t} \]  
(5.22)

The constrains given in Equations (5.19)-(5.21) and their interpretations are the same as Equations (5.4)-(5.6) and their interpretations. Equation (5.22) is similar to (5.12). The profit maximizing solution gives \[ p^* = \frac{r-b_2}{2-q} \quad \text{and} \quad x_m^* = \frac{qr-2b_2}{qt(2-q)} \] (see Appendix A). The profit for this case is \[ \Pi_{\text{Total}}^* = \frac{a_2q^4 - 2a_2q^3 - 2rb_2q + r^2q + 2b_2^2}{2qt(2-q)}. \] We additionally have \[ d^* = \frac{r-qr+b_2}{t(2-q)}. \] The market coverage is then equal to \[ k^* = \frac{2(r-b_2)}{t(2-q)}. \]

When \[ r \geq \frac{t(2-q)+2b_2}{2}, \] it is optimal for the monopolist to serve all consumers. If \[ r < \frac{t(2-q)+2b_2}{2}, \] some consumers are not served at the equilibrium. With imperfect
information, “no coupon segment” appears for all $r > 0$. Now, the firm is not giving coupons to some consumers; therefore, it cannot perform first degree price discrimination. There are some consumers who are charged the same price, $p$. In the free perfect information case, the firm has no incentive to charge the same price to two consumers as no two consumers have the same valuation for the product and the firm has perfect knowledge of their individual valuations. The firm is always better off if it offers a coupon to each consumer so that it can extract the consumers’ surplus in full.

**PROPOSITION 3:** Only if $r > \frac{2b_2}{q} + \sqrt{2a_2t(2-q)}$ and $r < \frac{t(2-q) - 2b_2}{2(1-q)}$, will a firm introduce the changing face value Internet coupons. When this is the case, the firm will distribute coupons to $\frac{2(qr - 2b_2)}{qt(2-q)}$ percent of the consumers, increasing the total profit by $\frac{2a_2tq^4 - 4a_2tq^3 + r^2q^2 - 4rbq + 4b_q^2}{2qt(2-q)}$. Otherwise, it is not profitable for the firm to introduce coupons.

### 5.3 Discussion of the Internet Couponing Models

In this section, we analyze the Internet couponing models in detail and gives comparative statics of the two models presented in the previous section.

#### 5.3.1 Profit and Price Analysis of the Internet Couponing Models

A summary of Internet couponing models for a monopolist, including the no coupon model, is given in Table 5. The most important result in this section is what happens to the price of the product when a firm chooses to provide Internet coupons, whether they are fixed face value or changing face value. This result is stated in the following proposition.
PROPOSITION 4: When an Internet couponing method is more profitable than providing no coupons, the monopolistic firm will increase its price to the regular segments of the market.

With Internet coupons, the firm always prefers to increase its price for high valuation consumers. However, there is another noteworthy result of Internet couponing models of the previous section. The price is a decreasing function of the customer profiling cost.

PROPOSITION 5: The improvements in customer profiling technologies that lead to cost reductions will make the monopolistic firm charge more for its product.

Proposition 5 means the firm is going to increase its price if the cost of customer profiling goes down. The advent of profiling technology will actually make the monopolist charge more for the product. This is true for both Internet couponing models (see Table 5). A follow up result is that the firm will also increase the face value of its coupon in the fixed coupon case with lower costs of profiling. Similarly, in the changing face value coupons, the firm will give more consumers coupons if the profiling technology becomes cheaper.

Even though the result of Proposition 5 has important managerial implications for couponing firms, the result given in this proposition should be taken with a grain of salt. Our model assumes that the consumers who buy without the coupon at price $p$ reveal themselves in the first period. This means that if the profiling technology becomes cheaper, the firm will try to target more consumers, i.e. it will be willing to give coupons to more consumers. In order to target more consumers the firm needs to make sure that there are more consumers left in the second period to target. This can be done by
reducing the size of the no coupon segment, \( d \). The only way to do this is to increase the price of the product for the no coupon segment.

### 5.3.2 Comparison of the Internet Couponing Models

The comparison of the prices, the size of the no coupon segments, and the market coverage of the two Internet models are given by the following proposition.

**PROPOSITION 6:** If \( q < \frac{r + b_2}{2r - b_2} \) we have,

1. The price with the fixed face value Internet coupons is greater than the price with the changing face value Internet coupons,
2. The size of the no coupon segment with the fixed face value Internet coupons is less than the size of the no coupon segment with the changing face value Internet coupons,
3. The size of the coupon segment with the fixed face value Internet coupons is larger than the size of the coupon segment with the changing face value Internet coupons,
4. The market coverage of the fixed face value Internet coupons is larger than the market coverage of the changing face value Internet coupons.

Which of the two Internet couponing models yields more profits depends heavily on the parameters of the models. Whether or not one couponing model generates more profit than the other cannot easily be established by an analytical formula. Conditions can be derived in terms of parameter values but they are rather complicated and hence do not allow easy interpretation\(^3\). Instead, in this subsection we give a numerical example to show how the profitability of the two models change with three important parameters of the model – the customer profiling cost \( b_2 \), the targeting accuracy cost \( a_2 \) and the

\(^3\) For example we have the following condition for the value of the common reservation price.

\[
r > \frac{2b_2q + 2b_2q^2 + 2\sqrt{-6a_2tq^6 + 15a_2tq^4 - 6a_2tq^4 - 3b_2tq^4 + 12b_2tq^4 - 15b_2tq^4 + 6b_2tq^4}}{2(2q - q)}
\]

If this condition holds, the changing face value Internet coupons will always provide higher profit than the fixed face value Internet coupons.
accuracy level $q$. Figure 12 shows what happens to the profits of the two models when these parameters change.

![Figure 12 Profit Comparison of the Internet Couponing Models.](image)

The vertical axis represents the profit difference between the changing face value Internet coupons and the fixed face value Internet coupons. A positive value indicates higher profits for the changing face value case. The parameter values are $r = 1$ and $t = 2$. The customer profiling cost coefficients, $b_2$, are 0.01 for the figure on the left, and 0.10 for the figure on the right. The targeting accuracy cost parameter $a_2$ increases from 0.01 to 0.15 from left to right (one value for each curve) in both figures in shown increments. The feasible values of $q$ for the first figure are in the interval [0.01, 1.00] and for the second figure they are in the interval [0.10, 1.00]. Outside these two intervals, coupon provision is not a feasible option.

The first result of interest is the effect of the targeting accuracy cost parameter $a_2$. Ceteris paribus, as $a_2$ increases it becomes harder and harder for the firm to choose the distribution of the changing face value Internet coupons over the fixed face value ones. The targeting accuracy has to be higher and higher to justify the increasing cost of accuracy. This means that if the targeting accuracy cost is comparatively high and the
accuracy levels are not sufficient, it will not be profitable to use the changing face value Internet coupons. The firm will stick with the fixed face value coupons. For some value of targeting accuracy cost, no accuracy level is large enough to make the changing face value Internet coupons a viable option.

The effect of the customer profiling cost $b_2$ is also important but not as significant as the targeting accuracy cost. If the customer profiling cost decreases, *ceteris paribus*, the changing face value Internet coupons will be more profitable.

As seen in Chapter 2 most of the major Internet couponing firms have technologies to provide the changing face value Internet coupons. However, these technologies are not widely used in practice. Our models suggest that the reason for this low usage might be high targeting costs and/or the lack of high accuracy.
Table 5 Comparison of the Internet Couponing Models

<table>
<thead>
<tr>
<th></th>
<th>No Coupons</th>
<th>Internet Coupons with Free Perfect Information</th>
<th>Internet Fixed Face Value Coupons</th>
<th>Internet Changing Face Value Coupons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price, $p^*$</td>
<td>$\frac{r}{2}$</td>
<td>$r$</td>
<td>$2r - b_2$</td>
<td>$\frac{r - b_2}{2 - q}$</td>
</tr>
<tr>
<td>Coupon Value, $f^*$</td>
<td>0</td>
<td>$xt$</td>
<td>$\frac{r - 2b_2}{3}$</td>
<td>$\left[0, \frac{qr - 2b_2}{qt(2 - q)}\right]$</td>
</tr>
<tr>
<td>No Coupon Segment, $d^*$</td>
<td>$\frac{r}{2t}$</td>
<td>0</td>
<td>$\frac{r + b_2}{3t}$</td>
<td>$\frac{r - qr + b_2}{t(2 - q)}$</td>
</tr>
<tr>
<td>Coupon Segment, $x^*$</td>
<td>0</td>
<td>$\frac{2r}{t}$</td>
<td>$\frac{r - 2b_2}{3t}$</td>
<td>$\frac{qr - 2b_2}{qt(2 - q)}$</td>
</tr>
<tr>
<td>Market Coverage, $k^*$</td>
<td>$\frac{r}{t}$</td>
<td>$\frac{r}{t}$</td>
<td>$\frac{2}{3t}(2r - b_2)$</td>
<td>$\frac{2(r - b_2)}{t(2 - q)}$</td>
</tr>
<tr>
<td>Firm’s Profit, $\Pi^*$</td>
<td>$\frac{r^2}{2t}$</td>
<td>$\frac{r^2}{t}$</td>
<td>$\frac{2}{3t}(r^2 - rb_2 + b_2^2)$</td>
<td>$\frac{a_4tq^4 - 2a_2tq^2 - 2rbq + r^2q + 2b_2^2}{2tq(2 - q)}$</td>
</tr>
<tr>
<td>Consumers’ Surplus</td>
<td>$\frac{r^2}{4t}$</td>
<td>0</td>
<td>$\frac{2r^2 - 2rb_2 + 5b_2^2}{9t}$</td>
<td>---</td>
</tr>
<tr>
<td>Total Welfare</td>
<td>$\frac{3r^2}{4t}$</td>
<td>$\frac{r^2}{t}$</td>
<td>$\frac{8r^2 - 8rb_2 + 11b_2^2}{9t}$</td>
<td>---</td>
</tr>
</tbody>
</table>
CHAPTER 6
COMPETITION OF NO, CONVENTIONAL AND INTERNET COUPONS

In this chapter, we investigate the competition of a no coupon, a conventional paper coupon or an Internet coupon user firm versus a no coupon, a conventional paper coupon or an Internet coupon user firm in a duopolistic setting. Since we analyze the cases where each firm is allowed to use only one type of coupon drop, there are a total of six possible cases as identified in Table 6. We analyze all of these six competitive cases using the models developed in Chapters 4 and 5. In the following sections we develop and solve each of these situations separately and then we look at the big picture and examine the more interesting decision of choosing a couponing strategy.

Table 6 Competitive Cases for Duopoly

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>Firm 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Coupon</td>
<td>Internet Coupon</td>
<td>Paper Coupon</td>
<td></td>
</tr>
<tr>
<td>No Coupon</td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
<td></td>
</tr>
<tr>
<td>Internet Coupon</td>
<td>Case 2</td>
<td>Case 4</td>
<td>Case 5</td>
<td></td>
</tr>
<tr>
<td>Paper Coupon</td>
<td>Case 3</td>
<td>Case 5</td>
<td>Case 6</td>
<td></td>
</tr>
</tbody>
</table>

6.1 Case 1: Review of Salop’s Model for a Duopoly

The first case is the no coupon case where two firms compete without coupons and the competition is modeled in the unit circle context. This case is analyzed in the literature (see Dewan et al. 2000 and Salop 1979); however, we are going to develop it here again to lay up the foundation for the later more involved models. Additionally, it will serve as a base case to compare with the later more involved cases.
In this and all games in this chapter we assume that two firms produce a homogenous product. Here we use the term homogenous very loosely; what we mean is that the products are technologically very similar and serve the same general need; however, their attributes are different, hence consumers have heterogeneous preferences towards the products. As the firms are similar we can assume that both have the same cost structure with zero marginal cost. Therefore they are identical from the game theoretical point of view. It is shown in the literature that the firms will differentiate their products as much as possible and locate as far as possible from each other on the unit circle (D’Aspremont et al. 1979). This is called the principle of maximal differentiation. We also make the following assumption regarding the common reservation value for the ideal product. This assumption is common in the literature (see Dewan et al. 2000, Salop 1979, Shaffer and Zhang 1995)

**ASSUMPTION 1: The common reservation price \( r \) is high enough so that all consumers are served at the equilibrium.**

Given assumption 1 the game between the firms is based on competing for the marginal consumers as shown in Figure 13. At this point we introduce a new notation to represent the locations of various marginal consumer types. As the games in the later sections become more and more complex our initial notation using \( d \) will be extremely confusing. In the figure the relative locations of marginal consumers are given by

\[
x_i \in [0, \frac{1}{2}]
\]

(in this case \( i = 1 \) but we will have more marginal consumer types in what follows), which is the distance of marginal consumer type \( i \) from Firm 1 on the unit circle.
Let $p_1$ and $p_2$ denote the prices of the firms. Then, given these prices the market shares of firms will be $2x_1$ and $1-2x_1$ as the market is totally covered. Given the prices, the following is true for the marginal consumer who is indifferent between buying from firm 1 and firm 2:

$$r - p_1 - x_1 t = r - p_2 - \left(\frac{1}{2} - x_1\right)t$$  \hspace{1cm} (6.1)

Or we can rewrite equation (6.1) as equation and eliminate the common reservation value from the equation altogether. It is noted from equation (6.2) the common reservation value is of no importance in a competitive situation as long as we make the assumption that the common reservation value is high enough that the market is fully covered at the equilibrium. Equation (6.2) implies that the total cost that the marginal consumer incurs for one product (which is the price she pays plus the fit cost she incurs) should be equal to the total cost she incurs for the other product. In the remaining of this work we will adopt this representation and find the location of marginal consumers through the use of their total costs rather than their net utility.
\[ p_1 + x_1 t = p_2 + \left( \frac{1}{2} - x_1 \right) t \]  

(6.2)

The demand function\(^1\) for firm 1 is  
\[ x_1 = \frac{p_2 - p_1 + \frac{t}{2}}{2t} \]  
Similarly, the demand function for firm 2 is  
\[ x_2 = \frac{p_1 - p_2 + \frac{t}{2}}{2t} \]  
The profit functions for the two firms are

\[ \Pi_1 = 2x_1 p_1 = 2 \left( \frac{p_2 - p_1 + \frac{t}{2}}{2t} \right) p_1 = \frac{p_1 p_2 - p_1^2}{t} + \frac{p_1}{2} \]  and

\[ \Pi_2 = 2(\frac{1}{2} - x_1) p_2 = 2 \left( \frac{p_1 - p_2 + \frac{t}{2}}{2t} \right) p_2 = \frac{p_2 p_1 - p_2^2}{t} + \frac{p_2}{2} \]

Given the profit functions, the problems these two firms need to solve can be formulated easily as follows.

\[ \text{Max}_{p_1} \left[ \frac{p_1 p_2 - p_1^2}{t} + \frac{p_1}{2} \right] \]  

(6.3)

\[ \text{Max}_{p_2} \left[ \frac{p_2 p_1 - p_2^2}{t} + \frac{p_2}{2} \right] \]  

(6.4)

We assume two firms simultaneously make their pricing decisions; therefore, in order to solve these two problems simultaneously, we find the first order conditions for both and solve them simultaneously.

F.O.C.

\(^1\) To be precise this is only one half of total demand. However we use the term “demand function” for this value as long as there is no confusion about what it means.
\[
\frac{d\Pi_1}{dp_1} = \frac{p_2 - 2p_1 + \frac{1}{2}}{t} = 0
\]

\[
\frac{d\Pi_2}{dp_2} = \frac{p_1 - 2p_2 + \frac{1}{2}}{t} = 0
\]

As \( \frac{d^2\Pi_1}{dp_1^2} = \frac{d^2\Pi_2}{dp_2^2} = - \frac{2}{t} < 0 \), the second order conditions for maximization are also satisfied. Solving both gives \( p_1^* = p_2^* = \frac{t}{2} \). Additionally, \( \Pi_1^* = \Pi_2^* = \frac{t}{4} \) and \( x_1^* = \frac{1}{4} \).

Furthermore, the market shares for the two firms are \( k_1^* = k_2^* = \frac{1}{2} \). This simple case constitutes a symmetric Nash equilibrium where each firm sets its price equal to each other and extracts exactly the same profit from the market. The following proposition summarizes this model. The following notation is used throughout this chapter to specify the equilibrium in prices: \( \langle p_1^*; p_2^*; f_1^*; f_2^*; k_1^*; k_2^*; \Pi_1^*; \Pi_2^* \rangle \). Note that \( f_1 \) and \( f_2 \) denote the face values of firm 1’s and firm 2’s coupons, which are zero in this case.

**PROPOSITION 7:** Under Assumption 1, the game between the two firms has a unique Nash equilibrium,

\[
\langle p_1^*; p_2^*; f_1^*; f_2^*; k_1^*; k_2^*; \Pi_1^*; \Pi_2^* \rangle = \left\langle \frac{t}{2}; \frac{t}{2}; 0; 0; \frac{1}{2}; \frac{1}{2}; \frac{t}{4}; \frac{t}{4} \right\rangle.
\]

Proof: To prove that the equilibrium given in the proposition is Nash equilibrium we need to show that neither of the two firms can deviate and charge a different price to improve its profit. Suppose firm 1 increases its price by \( \varepsilon \) given that the price of firm 2 is constant. It can be readily shown from the profit function given in the text that firm 1’s profit in this case will be \( \frac{t}{4} - \frac{\varepsilon^2}{t} \), which is less than that of the equilibrium. Alternatively if firm 1
decreases its price by \( \epsilon \) given that its competitor’s price is held constant, firm 1’s profit in this case will be \( \frac{t}{4} - \frac{\epsilon^2}{t} \), which is again less than that of the equilibrium. As two firms are identical, the same applies to firm 2; i.e. neither firm can deviate from the equilibrium prices and increase its profit. Therefore the equilibrium in prices given in the proposition is Nash. \( Q.E.D. \)

In the proceeding five sections we model Cases 2 – 6 of Table 6. The games between the firms are modeled in “stages” where decisions are made in sequence: first, pricing when regular prices are chosen, then, couponing when coupon face values and targeting strategies are chosen. To solve these sequential games we invoke backwards induction and find the Subgame Perfect Equilibrium (S.P.E.) for each case. These optimal solutions constitute a Nash noncooperative equilibrium (see Rao, 1991; Moorthy, 1985; Moorthy, 1988).

6.2 Case 2: Internet Coupon User Firm vs. No Coupon Firm

We model the game between an Internet coupon user firm and a no coupon user firm as a two-stage game. In the first stage both firms choose their prices simultaneously. Based on this, in the second stage the Internet firm chooses the face value of its fixed face value Internet coupon. To find the Subgame Perfect Equilibrium (S.P.E.) for this two-stage game we use backwards induction starting with stage 2 maximization problem of the Internet coupon user firm, called “Internet Couponing Game.” We designate the no coupon firm as Firm 1 and the Internet firm as Firm 2. Next section analyzes the second stage problem for Firm 2.
6.2.1 Stage 2: Internet Couponing Game

In this stage Firm 2 tries to set the face value of its coupon to maximize its additional profit from couponing. The prices that are set in the first stage are known with certainty by the two firms in the second stage.

Figure 14 depicts this two-stage model. Two marginal consumer types are given by \( x_i \in [0, \frac{1}{2}] \) \( (i = 1, 2) \), again, \( x_i \) is the distance of marginal consumer type \( i \) from Firm 1 on the unit circle.

Figure 14 Case 2: Internet Coupon vs. No Coupon

With the two marginal consumer types we will have three different regions (I, II and III). The consumers will have a distinct behavior in each of these regions depending on whether or not they get coupons. Table 7 gives detailed descriptions of these marginal consumers.

The following equations show indifference of each marginal consumer type mathematically.

\[
p_1 + x_i t = p_2 + \left( \frac{1}{2} - x_i \right) t - f_2 \tag{6.5}
\]
Table 7 Marginal Consumer Types and Descriptions for Case 2

<table>
<thead>
<tr>
<th>Marginal Consumer Type</th>
<th>Distance from Firm 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 2’s Internet coupon.</td>
</tr>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has none of the coupons.</td>
</tr>
</tbody>
</table>

\[
p_1 + x_2 t = p_2 + \left(\frac{1}{2} - x_2\right) t \tag{6.6}
\]

Solving for $x$’s we have,

\[
x_1 = \frac{p_2 - p_1 - f_2 + \frac{t}{2}}{2t} \tag{6.7}
\]

\[
x_2 = \frac{p_2 - p_1 + \frac{t}{2}}{2t} \tag{6.8}
\]

Having stated the descriptions and locations of marginal consumer types, we explain each region with the help of Table 8.

Table 8 Consumer Behavior in Different Regions for Case 2

<table>
<thead>
<tr>
<th>Region</th>
<th>Consumer behavior in the region</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buys from Firm 1.</td>
</tr>
<tr>
<td>II</td>
<td>Buys from Firm 2 if receives Firm 2’s coupon, otherwise buys from Firm 1.</td>
</tr>
<tr>
<td>III</td>
<td>Buys from Firm 2.</td>
</tr>
</tbody>
</table>

Now, we can calculate Firm 2’s second stage profits. Firm 2’s strategy is clear:

Target the consumers in region II with Internet coupons. Let $\Pi_2^2$ denote Firm 2’s stage 2 profits from couponing, then it can be formulated as follows:

\[
\Pi_2^2 = 2(p_2 - f_2 - b_2)(x_2 - x_1) \tag{6.9}
\]
Where $b_2$ represents the customer profiling cost as described in Chapter 5. Substituting the values of $x$’s from equations (6.7) and (6.8) into equation (6.9) gives,

\[
\Pi^*_2 = \frac{(p_2 - f_2 - b_2)f_2}{t} \quad \text{(6.10)}
\]

F.O.C.

\[
\frac{d\Pi^*_2}{df^*_2} = \frac{p_2 - b_2 - 2f_2}{t} = 0
\]

As \( \frac{d^2\Pi^*_2}{df^*_2} = -\frac{2}{t} < 0 \), the second order condition for maximization is also satisfied.

Solving the F.O.C. for $f^*_2$ gives \( f^*_2 = \frac{p_2 - b_2}{2} \). This is the face value of the Internet coupon that Firm 2 will distribute in Stage 2. Now we can go to Stage 1 and solve for the optimal prices.

6.2.2 Stage 1: Pricing Game

In the first stage the firms will set their prices to maximize their profits, anticipating the result of the second stage Internet couponing game. Let \( \Pi^*_1 \) and \( \Pi^*_2 \) denote the Stage 1 problems of Firm 1 and Firm 2 respectively, then these first stage problems for the firms can be formulated as follows:

\[
\Pi^*_1 = 2p_1x_1 \quad \text{(6.11)}
\]

\[
\Pi^*_2 = 2p_2\left(\frac{1}{2} - x_2\right) + 2(p_2 - f_2 - b_2)(x_2 - x_1) \quad \text{(6.12)}
\]

Substituting the values of $x$’s from equations (6.7), (6.8) and the value of \( f^*_2 \) into equations (6.11) and (6.12) gives,

\[
\Pi^*_1 = \frac{p_1(p_2 - 2p_1 + b_2 + t)}{2t} \quad \text{(6.13)}
\]
\[ \Pi_2 = \frac{2tp_2 - 3p_2^2 + 4p_1p_2 - 2b_2p_2 + b_2^2}{4t} \quad (6.14) \]

In order to solve the problems simultaneously, we find the first order conditions for both and solve them simultaneously.

F.O.C.

\[ \frac{d\Pi_1}{dp_1} = \frac{p_2 - 4p_1 + b_2 + t}{2t} = 0 \]

\[ \frac{d\Pi_2}{dp_2} = \frac{2p_1 - 3p_2 - b_2 + t}{2t} = 0 \]

As \( \frac{d^2\Pi_1}{dp_1^2} = -\frac{2}{t} < 0 \) and \( \frac{d^2\Pi_2}{dp_2^2} = -\frac{3}{2t} < 0 \), the second order conditions for maximization are also satisfied. Solving both first order conditions simultaneously gives

\[ p_1^* = \frac{2t + b_2}{5} \quad \text{and} \quad p_2^* = \frac{3t - b_2}{5}. \]

Hence, \( f_2^* = \frac{3t - 6b_2}{10} \). Additionally, \( x_1^* = \frac{2t + b_2}{10t} \) and

\[ x_2^* = \frac{7t - 4b_2}{20t}. \]

The no coupon firm’s market share is \( k_1^* = \frac{2t + b_2}{5t} \). The Internet firm’s market share is \( k_2^* = \frac{3t - b_2}{5t} \). The profits for the two firms are: \( \Pi_1^* = \frac{(2t + b_2)^2}{25t} \) and

\[ \Pi_2^* = \frac{27t^2 - 18tb_2 + 28b_2^2}{100t}. \]

The following proposition summarizes this model.

**PROPOSITION 8:** Under Assumption 1, the game between the two firms has a unique Subgame Perfect Equilibrium characterized as,

\[ \left\{ p_1^*, p_2^*; f_1^*, f_2^*; k_1^*, k_2^*; \Pi_1^*, \Pi_2^* \right\} = \left\{ \frac{2t + b_2}{5}, \frac{3t - b_2}{5}; 0, \frac{3t - 6b_2}{10}; \frac{2t + b_2}{5t}, \frac{3t - b_2}{5t}; \frac{(2t + b_2)^2}{25t}, \frac{27t^2 - 18tb_2 + 28b_2^2}{100t} \right\}. \]
6.3 Case 3: Conventional Paper Coupon User Firm vs. No Coupon Firm

We model the game between a conventional paper coupon user firm and a no coupon user firm as a two-stage game similar to Case 2. In the first stage both firms choose their prices simultaneously. Based on this, in the second stage the paper coupon user firm chooses the face value of its coupon and distributes it to all the consumers. We make another simplifying assumption at this point and assume that when a conventional firm distributes coupons all consumers who are willing to use it will get it. To find the Subgame Perfect Equilibrium (S.P.E.) for this two-stage game we use backwards induction starting with stage 2 maximization problem of the paper coupon user firm, called “Paper Couponing Game.” We designate the conventional paper coupon user firm as Firm 1 and the no coupon user firm as Firm 2. Next section analyzes the second stage problem for Firm 1.

6.3.1 Stage 2: Paper Couponing Game

In this stage Firm 1 tries to set the face value of its coupon to maximize its additional profit from couponing. The prices that are set in the first stage are known with certainty by the two firms in the second stage. In this case, contrast to the previous case; the conventional paper coupon user firm cannot target its coupons. As we explained in detail in Chapter 4, these coupons are mass media distributed paper coupons. Therefore, in the second stage Firm 1 will cannibalize some its own loyal consumers who would buy from Firm 1 by giving them coupons. The trade-off between this “cannibalization effect” and additional profits from those who switch to Firm 1 due to the coupon will determine the faith of Firm’1 paper coupon and its face value.
Figure 15 exhibits this two-stage model, where $x_i \in [0, \frac{1}{2}]$ ($i=1,2,3$) represents the distance of marginal consumer type $i$ from Firm 1 as defined earlier.

![Diagram showing two stages: Firm 1: Paper and Firm 2: No Coupon]

Figure 15 Case 3: Paper Coupon vs. No Coupon

With the three marginal consumer types we will have four different regions (I, II, III and IV). Table 9 gives detailed descriptions of these marginal consumers.

Table 9 Marginal Consumer Types and Descriptions for Case 3

<table>
<thead>
<tr>
<th>Marginal Consumer Type</th>
<th>Distance from Firm 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>Indifferent between using or not using Firm 1’s paper coupon.</td>
</tr>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has none of the coupons.</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>Indifferent between buying from Firm 1 using Firm 1’s coupon or buying from Firm 2.</td>
</tr>
</tbody>
</table>

Before we explain how the location of each of these marginal consumer types is derived, let us briefly remind the basic model for conventional paper coupons that we developed in Chapter 4. We stated earlier that there is a disutility associated with using the paper coupon. We let $\kappa_i$ to represent a consumer $i$’s disutility of using the paper coupon. We assumed that consumers who are closer to their ideal product would have
less incentive to use coupons. Therefore the disutility of using coupons will decrease as
we move away from the point where the product is marketed. The decrease per unit
distance of circle is designated by \( \alpha \). Then we let \( Z \) to represent the maximum disutility
of using a paper coupon for all the consumers. Since the maximum distance is \( \frac{1}{2} \) the
minimum disutility of using coupons for any consumer will be \( Z - \frac{\alpha}{2} \). Hence, the
disutility of using coupons is described by \( \kappa = Z - \alpha d \), where \( d \) denotes the distance
from the product a coupon provided for. For instance, the term \( Z - x_1 \alpha \) in equation (6.15)
represents the disutility of using the paper coupon for the consumer located at point \( x_1 \).
The following equations show the indifference of each marginal consumer type
mathematically.

\[
f'_i = Z - x_i \alpha \quad (6.15)
\]

\[
p_1 + x_2 t = p_2 + \left( \frac{1}{2} - x_2 \right) t \quad (6.16)
\]

\[
p_1 + x_3 t - f_i + (Z - x_3 \alpha) = p_2 + \left( \frac{1}{2} - x_1 \right) t \quad (6.17)
\]

Solving for \( x' \)s we have,

\[
x_1 = \frac{Z - f_i}{\alpha} \quad (6.18)
\]

\[
x_2 = \frac{p_2 - p_1 + \frac{t}{2}}{2t} \quad (6.19)
\]

\[
x_3 = \frac{p_2 - p_1 + f_i - Z + \frac{t}{2}}{2t - \alpha} \quad (6.20)
\]
Having stated the descriptions and locations of marginal consumer types, we can explain each region with the help of Table 10.

Table 10 Consumer Behavior in Different Regions for Case 3

<table>
<thead>
<tr>
<th>Region</th>
<th>Consumer behavior in the region</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buys from Firm 1 but does not use Firm 1’s coupon.</td>
</tr>
<tr>
<td>II</td>
<td>Buys from Firm 1 and uses Firm 1’s coupon (would buy from Firm 1 even without a the coupon).</td>
</tr>
<tr>
<td>III</td>
<td>Buys from Firm 1 and uses Firm 1’s coupon (induced to buying because of the coupon).</td>
</tr>
<tr>
<td>IV</td>
<td>Buys from Firm 2.</td>
</tr>
</tbody>
</table>

Now, we can calculate Firm 1’s profit in the second stage. Let $\Pi_1^2$ denote Firm 1’s stage 2 profits from couponing, then it can be formulated as follows:

$$\Pi_1^2 = 2(p_1 - f_1) (x_3 - x_2) - 2f_1 (x_3 - x_1) - a$$  \hspace{1cm} (6.21)

Where $a$ represents the cost of coupon distribution as defined in Chapter 4. It is fixed in this Chapter as we assume that the coupon distribution intensity ($\lambda$) is one. The first term in this profit figure is the additional profit due to the coupon while the second term is the cost of “cannibalization effect.” Substituting the values of $x$’s from equations (6.18), (6.19) and (6.20) into equation (6.21) and taking the first order condition gives, F.O.C.

$$\frac{d\Pi_1^2}{df_1} = \frac{4\alpha p_1 - 2\alpha p_2 - t\alpha + 4tZ - 8tf_1}{(2t - \alpha)\alpha} = 0$$

As $\frac{d^2\Pi_1^2}{df_1^2} = -\frac{8t}{(2t - \alpha)\alpha} < 0$, the second order condition for maximization is also satisfied.

Solving the F.O.C. for $f_1$ gives $f_1^* = \frac{4\alpha p_1 - 2\alpha p_2 - t\alpha + 4tZ}{8t}$. This is the face value of the
paper coupon that Firm 1 will distribute in Stage 2. Now we can go to Stage 1 and solve for the optimal prices.

6.3.2 Stage 1: Pricing Game

In the first stage the firms will set their prices to maximize their profits, anticipating the result of the second stage Internet couponing game. Let \( \Pi_1 \) and \( \Pi_2 \) denote the Stage 1 problems of Firm 1 and Firm 2 respectively, then these first stage problems for the firms can be formulated as follows:

\[
\Pi_1^1 = 2p_1x_1 + 2(p_1 - f_1)(x_1 - x_i) - a \\
\Pi_2^1 = 2p_2\left(\frac{1}{2} - x_3\right)
\]

Substituting the values of \( x \)'s from equations (6.18), (6.19) and (6.20) into equations (6.22) and (6.23) and taking the first order condition gives,

\[
F.O.C. \\
\frac{d\Pi_1^1}{dp_1} = \frac{2p_2 - 4p_1 + t}{2t} = 0 \\
\frac{d\Pi_2^1}{dp_2} = \frac{8tp_1 - 4(4t - \alpha)p_2 - 4\alpha p_1 + 4tZ + 4t^2 - 3t\alpha}{4t(2t - \alpha)} = 0
\]

As \( \frac{d^2\Pi_1^1}{dp_1^2} = -\frac{2}{t} < 0 \) and \( \frac{d^2\Pi_1^1}{dp_2^2} = -\frac{(4t - \alpha)}{t(2t - \alpha)} < 0 \), the second order conditions for maximization are also satisfied. Solving both first order conditions simultaneously gives

\[
p_1^* = \frac{t(12t - 5\alpha + 4Z)}{4(6t - \alpha)} \quad \text{and} \quad p_2^* = \frac{t(3t - 2\alpha + 2Z)}{6t - \alpha}. \quad \text{Hence,} \quad f_1^* = \frac{Z}{2}. \quad \text{Additionally,}
\]

\[
x_1^* = \frac{Z}{2\alpha}, \quad x_2^* = \frac{12t - 5\alpha + 4Z}{8(6t - \alpha)} \quad \text{and} \quad \text{Firm 1’s market share is} \quad k_1^* = \frac{12t^2 - 5t\alpha - 8tZ + 2\alpha Z}{2(6t - \alpha)(2t - \alpha)}.
\]
Firm 2’s market share is \( k_2^* = \frac{12t^2 - 11t\alpha + 8tZ - 2aZ + 2\alpha^2}{2(6t - \alpha)(2t - \alpha)} \). The profits for the two firms are:

\[
\Pi_1^* = -\frac{192\alpha Z t^3 + 128\alpha^2 Z t^2 - 20\alpha^3 Z t + 288Z^2 t^3 - 176\alpha Z^2 t^2 + 24\alpha^2 Z^2 t}{8\alpha(6t - \alpha)^2(2t - \alpha)} + \frac{144\alpha t^4 - 120\alpha^2 t^3 + 25\alpha^3 t^2 - 576\alpha t^3 + 480\alpha^2 t^2 - 112\alpha^3 t + 8\alpha^4}{8\alpha(6t - \alpha)^2(2t - \alpha)}.
\]

and \( \Pi_2^* = \frac{t(3-2\alpha + 2Z)(12t^2 - 11t\alpha + 8tZ - 2aZ + 2\alpha^2)}{2(6t - \alpha)^2(2t - \alpha)} \). The following proposition summarizes this model.

**PROPOSITION 9**: Under Assumption 1, the game between the two firms has a unique Subgame Perfect Equilibrium characterized as,

\[
\left\{ p_1^*, p_2^*; f_1^*, f_2^*; k_1^*, k_2^*; \Pi_1^*, \Pi_2^* \right\} = \begin{cases} 
\left( \frac{t(12t - 5\alpha + 4Z)}{4(6t - \alpha)}, \frac{t(3-2\alpha + 2Z)}{6t - \alpha}; 0, \frac{12t^2 - 5t\alpha - 8tZ + 2aZ}{4(6t - \alpha)(2t - \alpha)} \right), \\
\left( \frac{12t^2 - 11t\alpha + 8tZ - 2aZ + 2\alpha^2}{2(6t - \alpha)(2t - \alpha)}; \right.
\end{cases}
\]

\[
\left( \frac{t(3-2\alpha + 2Z)(12t^2 - 11t\alpha + 8tZ - 2aZ + 2\alpha^2)}{2(6t - \alpha)^2(2t - \alpha)} \\
\frac{-192\alpha Z t^3 + 128\alpha^2 Z t^2 - 20\alpha^3 Z t + 288Z^2 t^3 - 176\alpha Z^2 t^2 + 24\alpha^2 Z^2 t}{8\alpha(6t - \alpha)^2(2t - \alpha)} + \frac{144\alpha t^4 - 120\alpha^2 t^3 + 25\alpha^3 t^2 - 576\alpha t^3 + 480\alpha^2 t^2 - 112\alpha^3 t + 8\alpha^4}{8\alpha(6t - \alpha)^2(2t - \alpha)}, \\
\right)
\]

**6.4 Case 4: Internet Coupon User Firm vs. Internet Coupon User Firm**

In this section we model the competition between an Internet coupon user firm and an Internet coupon user firm. The game between these two firms is modeled as a two-stage game. In the first stage both firms determine their prices \((p_1 \text{ and } p_2)\) and make these prices public. In the second stage both firms decide the face values of their Internet coupons. As it is possible for the firms using the Internet coupon to target the coupon to
select consumers, at this stage the firms also decide on their targeting strategy. To find
the Subgame Perfect Equilibrium (S.P.E.) for this two-stage game we use backwards
induction starting with stage 2 maximization problems called “Internet Coupon Targeting
Game.” Before we look at the second stage problems let us identify marginal consumer

types for this case. In this game we have up to four types of marginal consumers. Figure
16 illustrates the possible types of marginal consumers: \( x_i \in [0, \frac{1}{2}] \) (\( i = 1, 2, 3, 4 \)) as defined
earlier.

**Table 11 Marginal Consumer Types and Descriptions for Case 4**

<table>
<thead>
<tr>
<th>Marginal Consumer Type</th>
<th>Distance from Firm 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x_1 )</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 2’s coupon but not Firm 1’s.</td>
</tr>
<tr>
<td>2</td>
<td>( x_2 )</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has both the Firm 2’s and Firm 1’s coupons.</td>
</tr>
<tr>
<td>3</td>
<td>( x_3 )</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has none of the coupons.</td>
</tr>
<tr>
<td>4</td>
<td>( x_4 )</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 1’s coupon but not Firm 2’s.</td>
</tr>
</tbody>
</table>
With four marginal consumer types we will have up to five regions labeled with
Roman numerals I through V. Table 11 gives detailed descriptions of these marginal
consumers.

The following equations show indifference of each marginal consumer type
mathematically.

\[ p_1 + x_t = p_2 + \left(\frac{1}{2} - x_i\right)t - f_2 \]  \hspace{1cm} (6.24)

\[ p_1 + x_2t - f_i = p_2 + \left(\frac{1}{2} - x_2\right)t - f_2 \]  \hspace{1cm} (6.25)

\[ p_1 + x_3t = p_2 + \left(\frac{1}{2} - x_3\right)t \]  \hspace{1cm} (6.26)

\[ p_1 + x_4t - f_i = p_2 + \left(\frac{1}{2} - x_4\right)t \]  \hspace{1cm} (6.27)

Solving for \( x' \)s we have,

\[ x_1 = \frac{p_2 - p_1 - f_i + \frac{t}{2}}{2t} \]  \hspace{1cm} (6.28)

\[ x_2 = \frac{p_2 - p_1 + f_i - f_2 + \frac{t}{2}}{2t} \]  \hspace{1cm} (6.29)

\[ x_3 = \frac{p_2 - p_1 + \frac{t}{2}}{2t} \]  \hspace{1cm} (6.30)

\[ x_4 = \frac{p_2 - p_1 + f_i + \frac{t}{2}}{2t} \]  \hspace{1cm} (6.31)

Having stated the descriptions and locations of marginal consumer types, we can
explain each region with the help of Table 12. In the next section we solve the second
stage couponing problems for Firm 1 and Firm 2.
Table 12 Consumer Behavior in Different Regions for Case 4

<table>
<thead>
<tr>
<th>Region</th>
<th>Consumer behavior in the region</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buys from Firm 1.</td>
</tr>
<tr>
<td>II</td>
<td>Buys from Firm 2 if receives only Firm 2’s coupon, otherwise buys from Firm 1.</td>
</tr>
<tr>
<td>III</td>
<td>Buys from Firm 2 if receives Firm 2’s coupon, otherwise buys from Firm 1.</td>
</tr>
<tr>
<td>IV</td>
<td>Buys from Firm 1 if receives only Firm 1’s coupon, otherwise buys from Firm 2.</td>
</tr>
<tr>
<td>V</td>
<td>Buys from Firm 2.</td>
</tr>
</tbody>
</table>

6.4.1 Stage 2: Internet Coupon Targeting Game

The profits of both firms in any given region depend on the targeting strategy followed by the two firms in that region. Therefore we first need to find out whether or not the firms have a clear targeting strategy to follow in each region. Looking at Table 12 and Table 13, it is clearly seen that none of the firms is willing to target consumers in regions I and V. These consumers are not willing to switch and they are loyal to one of the firms.

Consider a representative consumer in region III. This consumer prefers Firm 1’s product and willing to pay price $p_1$ when she does not have any coupons on hand. However, if she has both the firms’ coupons she prefers Firm 2. As a matter of fact, only way does this consumer buy from Firm 2 is if she receives Firm 2’s targeted coupon. It does not matter whether she receives Firm 1’s coupon in addition to Firm 2’s; as long as she has Firm 2’s coupon she prefers Firm 2, otherwise buys from Firm 1. Therefore targeting this region with probability one emerges as a dominant strategy for Firm 2. Knowing this Firm 1 will not target the consumers in this region, as targeting does not improve its profits in this region. As a matter of fact Firm 1 is better off by not targeting this region since targeting costs money. To summarize, Firm 1 does not target coupons in any of regions I, III and V whereas Firm 2 only targets in region III.
The targeting decisions for regions II and IV are not as clear as the other three regions. Looking at Table 13, we see that regions II and IV are symmetric. As we will show next, in both of these regions no pure strategy Nash equilibrium exists and we will need to find the unique mixed strategy Nash equilibria for these two regions. Consider region II. In this region Firm 1 does not want to target coupons as consumers are predisposed to buy from Firm 1. However, Firm 1 has to target if Firm 2 targets to protect its market share. Shaffer and Zhang (1995) call Firm 1’s behavior in region II “defensive targeting.” The situation is little different for Firm 2. It only wants to target when Firm 1 does not target. This behavior is called “offensive targeting” by Shaffer and Zhang (1995).

In summary, Firm 1 wants to do exactly what Firm 2 does while Firm 2 wants to do exact opposite of what Firm 1 does in region II. Hence, no pure strategy Nash equilibrium emerges in region II. The argument for region IV is exactly the same with roles of Firm 1 and Firm 2 reversed. The two firms have to exercise some “defensive targeting” to protect themselves from the other’s “offensive targeting.” In order to find the expected profits in regions II and IV, we need to know how often each firm targets in these regions. This is what the mixed strategy equilibrium implies.

In any mixed strategy Nash equilibrium a firm is going to mix its pure strategies with certain probabilities so that the rival firm is indifferent between the rival firm’s pure strategies. That is, the mixing probabilities of one firm should make the other firm indifferent between its pure strategies and vice versa.

Let $Pr_j^i$ denote targeting probability of Firm $j$ in region $i$. For region II $Pr_1^{II}$ and $Pr_2^{II}$ must satisfy the following two equations:
Table 13 Payoffs (Given as Firm 1’s payoff, Firm 2’s payoff) for Case 4

<table>
<thead>
<tr>
<th>Region</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Target</td>
<td>Don't Target</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
Pr_i^{II} \left[ 2(p_1 - f_1 - b_2)(x_2 - x_i) \right] + (1 - Pr_i^{II}) \left[ 2(p_1 - f_1 - b_2)(x_2 - x_i) \right] = Pr_i^{II} 0 + (1 - Pr_i^{II}) \left[ 2p_i(x_2 - x_i) \right],
\]

(6.32)

\[
Pr_i^{II} \left[ (-2b_2)(x_2 - x_1) \right] + (1 - Pr_i^{II}) \left[ 2(p_2 - f_2 - b_2)(x_2 - x_1) \right] = Pr_i^{II} 0 + (1 - Pr_i^{II}) 0
\]

(6.33)

Simplifying equations (6.32) and (6.33) gives,
\[ p_1 - f_1 - b_2 = (1 - Pr^{II}_1) p_1, \quad (6.34) \]

\[ Pr^{II}_1 (-b_2) + (1 - Pr^{II}_1) (p_2 - f_2 - b_2) = 0. \quad (6.35) \]

Solving equations (6.34) and (6.35) simultaneously we have:

\[ Pr^{II}_2 = \frac{f_1 + b_2}{p_1}, \quad (6.36) \]

\[ Pr^{II}_1 = \frac{p_2 - f_2 - b_2}{p_2 - f_2}. \quad (6.37) \]

For region IV, as it is symmetric to region II we have the following probabilities:

\[ Pr^{IV}_1 = \frac{f_2 + b_2}{p_2}, \quad (6.38) \]

\[ Pr^{IV}_2 = \frac{p_1 - f_1 - b_2}{p_1 - f_1}. \quad (6.39) \]

Given these probabilities we can go ahead and calculate the expected profits for each firm in regions II and IV and finally for the second stage targeting game. However, the fact that each firm is indifferent between mixing or playing one its own pure strategies with probability one relieves us from calculating the expected profits.

Therefore, Firm 1’s expected profit in region II is what it would get if it targets all the time, \( 2(p_1 - f_1 - b_2)(x_2 - x_1) \) and Firm 2’s expected profit in region II is what it would get if it does not target at all, 0. Similarly in region IV Firm 1’s expected is what it would get if it does not target at all, 0 and Firm 2’s expected profit is what it would get if it targets all the time, \( 2(p_2 - f_2 - b_2)(x_4 - x_3) \). Given this stage 2 profits of the two firms are:

\[ \Pi_1^2 = 2p_1x_1 + 2(p_1 - f_1 - b_2)(x_2 - x_1) \quad (6.40) \]

\[ \Pi_2^2 = 2(p_2 - f_2 - b_2)(x_4 - x_2) + 2p_2\left(\frac{1}{2} - x_4\right) \quad (6.41) \]
Substituting the values of $x$’s from equations (6.28) to (6.31) into equations (6.40) and (6.41) and taking the first order derivatives gives the following first order conditions:

F.O.C.

$$\frac{d\Pi_1^2}{df_1} = \frac{p_1 - b_x - 2f_1}{t} = 0$$

$$\frac{d\Pi_2^2}{df_2} = \frac{p_2 - b_x - 2f_2}{t} = 0$$

As $\frac{d^2\Pi_1^2}{df_1^2} = \frac{d^2\Pi_2^2}{df_2^2} = -\frac{2}{t} < 0$, the second order conditions for maximization are also satisfied. Solving the F.O.C.’s for $f_1$ and $f_2$ gives $f_1^* = \frac{p_1 - b_2}{2}$ and $f_2^* = \frac{p_2 - b_2}{2}$. These are the face values of the Internet coupons that Firms 1 and 2 would target and distribute in Stage 2. Now we can go to Stage 1 and solve for the optimal prices of both firms.

**6.4.2 Stage 1: Pricing Game**

In the first stage the firms will set their prices to maximize their profits, anticipating the result of the second stage Internet couponing game. The first stage problems for the both firms can be formulated as follows by substituting the values of $x$’s from equations (6.28) to (6.31) as well as the values of $f_1^*$ and $f_2^*$ into equations (6.40) and (6.41):

$$\Pi_1^1 = \frac{2tp_1 - 3p_1^2 + 2p_1p_2 + b_2^2}{4t}, \quad (6.42)$$

$$\Pi_2^1 = \frac{2tp_2 - 3p_2^2 + 2p_1p_2 + b_2^2}{4t}. \quad (6.43)$$

In order to solve the problems simultaneously, we find the first order conditions for both and solve them simultaneously.
F.O.C.
\[
\frac{d\Pi_1^1}{dp_1} = \frac{t - 3p_1 + p_2}{2t} = 0
\]
\[
\frac{d\Pi_1^1}{dp_2} = \frac{t - 3p_2 + p_1}{2t} = 0
\]

As \(\frac{d^2\Pi_1^1}{dp_1^2} = \frac{d^2\Pi_1^1}{dp_2^2} = -\frac{3}{2t} < 0\), the second order conditions for maximization are also satisfied. Solving both first order conditions simultaneously gives \(p_1^* = p_2^* = \frac{t}{2}\) and
\[
f_1^* = f_2^* = \frac{t - 2b_2}{4}\]. Additionally, \(x_1^* = \frac{t + 2b_2}{8t}, \ x_2^* = x_3^* = \frac{1}{4}, \ x_4^* = \frac{3t - 2b_2}{8t}\). The market shares are \(k_1^* = k_2^* = \frac{1}{2}\). The profits for the two firms are \(\Pi_1^* = \Pi_2^* = \frac{3t^2 + 4b_2^2}{16t}\).

The following proposition summarizes this model.

**PROPOSITION 10:** Under Assumption 1, the game between the two firms has a unique Subgame Perfect Equilibrium characterized as,

\[
\left\{p_1^*, p_2^*; f_1^*, f_2^*; k_1^*, k_2^*; \Pi_1^*, \Pi_2^* \right\}
\]

\[
= \left\{t, \ t; \ t - \frac{2b_2}{4}, \ t - \frac{2b_2}{4}; \ 1, \frac{1}{2}, \frac{1}{2}, \frac{3t^2 + 4b_2^2}{16t}, \frac{3t^2 + 4b_2^2}{16t} \right\}.
\]

**6.5 Case 5: Internet Coupon User Firm vs. Conventional Paper Coupon User Firm**

In this section we model the competition between an Internet coupon user firm and a conventional paper coupon user firm. The game between these two firms is modeled as a three-stage sequential game. In the first stage both firms determine their prices. In the second stage the firm using the conventional paper coupon decides on the face value of its coupon and distributes it. In the third stage the Internet coupon user firm decides whether or not to use its Internet coupon. The rationale behind is as follows. As
we have described in Chapter 2, the Internet coupons are very flexible and the delivery is almost instantaneous. Therefore the firm using the Internet firm can wait and see the paper coupon offering of the other firm and respond accordingly. In order to analyze all possible strategies for the two firms we will start with identifying a set of marginal consumer types. In this game we have up to five types of marginal consumers. We illustrate all the possible types of marginal consumers with the help of Figure 17.

In the figure, the relative locations of marginal consumer types are given by
\[ x_i \in [0, \frac{1}{2}] \ (i = 1, 2, 3, 4, 5) \]
defined as the distance of marginal consumer type \( i \) to the paper coupon distributing firm. With five marginal consumer types we will have up to six different regions (I, II,..,VI). The consumers will have a distinct behavior in each of these regions depending on the number of coupons they get. Before we explain how this works we need to state what each of the five marginal consumer types means. Table 14 gives detailed descriptions of these marginal consumers.

![Figure 17 Case 5: Paper Coupon vs. Internet Coupon](image-url)

The following equations show indifference of each marginal consumer type mathematically.
Table 14 Marginal Consumer Types and Descriptions for Case 5

<table>
<thead>
<tr>
<th>Marginal Consumer Type</th>
<th>Distance from Firm 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>Indifferent between using or not using the Firm 1’s paper coupon.</td>
</tr>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 2’s coupon but not Firm 1’s.</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has both the Firm 2’s and Firm 1’s coupons.</td>
</tr>
<tr>
<td>4</td>
<td>$x_4$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has none of the coupons.</td>
</tr>
<tr>
<td>5</td>
<td>$x_5$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 1’s coupon but not Firm 2’s.</td>
</tr>
</tbody>
</table>

\[
f_1 = Z - x_1 \alpha \quad (6.44)
\]

\[
p_1 + x_2 t = p_2 + \left( \frac{1}{2} - x_2 \right) t - f_2 \quad (6.45)
\]

\[
p_1 + x_3 t - f_1 + \left( Z - x_3 \alpha \right) = p_2 + \left( \frac{1}{2} - x_3 \right) t - f_2 \quad (6.46)
\]

\[
p_1 + x_4 t = p_2 + \left( \frac{1}{2} - x_4 \right) t \quad (6.47)
\]

\[
p_1 + x_5 t - f_1 + \left( Z - x_5 \alpha \right) = p_2 + \left( \frac{1}{2} - x_5 \right) t \quad (6.48)
\]

Solving for $x$’s we have,

\[
x_1 = \frac{Z - f_1}{\alpha} \quad (6.49)
\]

\[
x_2 = \frac{p_2 - p_1 - f_2 + \frac{t}{2}}{2t} \quad (6.50)
\]

\[
x_3 = \frac{p_2 - p_1 + f_1 - f_2 - Z + \frac{t}{2}}{2t - \alpha} \quad (6.51)
\]
\[ x_4 = \frac{p_2 - p_1 + \frac{r}{2}}{2t} \]  
(6.52)

\[ x_5 = \frac{p_2 - p_1 + f_1 - Z + \frac{r}{2}}{2t - \alpha} \]  
(6.53)

Having stated the descriptions and locations of marginal consumer types, we can explain each region with the help of Table 15.

<table>
<thead>
<tr>
<th>Region</th>
<th>Consumer behavior in the region</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buys from Firm 1 (does not use Firm 1’s coupon even if receives).</td>
</tr>
<tr>
<td>II</td>
<td>Buys from Firm 1 and uses Firm 1’s coupon if receives.</td>
</tr>
<tr>
<td>III</td>
<td>Buys from Firm 2 if receives only Firm 2’s coupon, otherwise buys from Firm 1 (uses the coupons).</td>
</tr>
<tr>
<td>IV</td>
<td>Buys from Firm 2 if receives Firm 2’s coupon, otherwise buys from Firm 1 (uses the coupons).</td>
</tr>
<tr>
<td>V</td>
<td>Buys from Firm 1 if receives only Firm 1’s coupon, otherwise buys from Firm 2 (uses the coupons).</td>
</tr>
<tr>
<td>VI</td>
<td>Buys from Firm 2 (uses the coupon).</td>
</tr>
</tbody>
</table>

Now, we can calculate each firm’s payoffs in each of the six regions. These payoffs are given in Table 16. In each box Firm 1’s payoff is given followed by a comma and Firm 2’s payoff.

Firm 1 has only two choices: distribute or does not distribute coupons to consumers in all six regions. However, if Firm 1 does not distribute coupons this case will reduce to Case 2 of Table 6. Therefore, in this case we assume that Firm 1 strictly distributes coupons. Firm 2 can decide on its targeting strategy for each region separately. Given the payoff table we can find the expected profits of both firms. We can start with Firm 1’s profits as we can determine Firm 2’s best strategy given what Firm 1 does in the second stage, this is possible as the couponing decision of Firm 2 in
stage 3 follows that of Firm 1 in stage 2. Once Firm 2 knows what Firm 1 is doing, it has dominant strategy for all six regions in the third stage of the game as follows: Firm 2’s dominant strategy is to target regions IV and V all the time and do not target any other region.

### Table 16 Payoffs (Given as Firm 1’s payoff, Firm 2’s payoff) for Case 5

<table>
<thead>
<tr>
<th>Region</th>
<th>Firm 2</th>
<th>Firm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribute</td>
<td>Do not distribute</td>
</tr>
<tr>
<td>I</td>
<td>Target</td>
<td>2(p_1x_1 - ax_1), -2b_2x_1</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>2(p_1x_1 - ax_1), 0</td>
</tr>
<tr>
<td>II</td>
<td>Target</td>
<td>2(p_1 - f_1 - a)(x_2 - x_1), -2b_2(x_2 - x_1)</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>2(p_1 - f_1 - a)(x_2 - x_1), 0</td>
</tr>
<tr>
<td>III</td>
<td>Target</td>
<td>2(p_1 - f_1 - a)(x_3 - x_2), -2b_2(x_3 - x_2)</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>2(p_1 - f_1 - a)(x_3 - x_2), 0</td>
</tr>
<tr>
<td>IV</td>
<td>Target</td>
<td>-2a(x_4 - x_3), 2(p_2 - f_2 - b_2)(x_4 - x_3)</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>2(p_1 - f_1 - a)(x_4 - x_3), 0</td>
</tr>
<tr>
<td>V</td>
<td>Target</td>
<td>-2a(x_5 - x_4), 2(p_2 - f_2 - b_2)(x_5 - x_4)</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>2(p_1 - f_1 - a)(x_5 - x_4), 0</td>
</tr>
<tr>
<td>VI</td>
<td>Target</td>
<td>-2a\left(\frac{1}{2} - x_3\right), 2(p_2 - f_2 - b_2)(\frac{1}{2} - x_5)</td>
</tr>
<tr>
<td></td>
<td>Don’t target</td>
<td>-2a\left(\frac{1}{2} - x_3\right), 2p_2(\frac{1}{2} - x_5)</td>
</tr>
</tbody>
</table>
6.5.1 Stage 3: Internet Couponing Game of Firm 2

We can calculate the profits of Firm 2 in the third stage by using Table 16 and its dominant strategy. Let \( \Pi^3_2 \) denote the third stage profits of Firm 2. We have the following for this value:

\[
\Pi^3_2 = 2(p_2 - f_2 - b_2)(x_3 - x_1) + 2p_2\left(\frac{1}{2} - x_3\right).
\] (6.54)

Substituting the values of \( x's \) from equations (6.50) to (6.53) into equation (6.54) gives,

\[
\Pi^3_2 = \frac{(t - \alpha)p_2 + 2p_1p_2 - 2p_2f_1 + 2p_2Z + 2(p_2 - b_2)f_2 - 2f_2^2}{2t - \alpha}.
\] (6.55)

To maximize this value we find the first order condition.

F.O.C.

\[
\frac{d\Pi^3_2}{df_2} = \frac{2(p_2 - b_2) - 4f_2}{2t - \alpha} = 0
\]

As \( \frac{d^2\Pi^3_2}{df_2^2} = -\frac{4}{2t - \alpha} < 0 \), the second order condition for maximization is also satisfied.

Solving the F.O.C. for \( f_2^{*} \) gives \( f_2^{*} = \frac{p_2 - b_2}{2} \). This is the face value of the Internet coupon that Firm 2 will distribute in Stage 3. This result indicates that Firm 2’s optimal coupon face value does not depend on the face value of Firm 1’s coupon. However, the change in the value of \( f_1 \) lends itself in the values of \( x's \), hence in the profits of Firm 2 indirectly. Even though Firm 2 does not change its coupon’s face value it changes its targeting range in accordance with the changes in the face value of Firm 1’s coupon. In the next section we formulate the second stage problem of Firm 1 and find the optimal face value of its paper coupon.
6.5.2 Stage 2: Paper Couponing Game of Firm 1

In this stage we make some simplifying assumptions that we made in Case 3 of Table 6 and assume that the distribution intensity (\( \lambda \)) of Firm 1’s mass media distributed paper coupon is one. That is, Firm 1’s paper coupons will reach all the consumers that are interested in using them. Therefore at this stage Firm 1 is concerned only with the face value of its coupon. The profits of Firm 1 at this stage, denoted by \( \Pi_1^2 \), can be found as,

\[
\Pi_1^2 = 2p_1x_1 + 2(p_1 - f_1)(x_3 - x_1) - a.
\]  

(6.56)

Substituting the values of corresponding \( x \)'s from equations (6.50) to (6.53) and the value of \( f_2 \) into equation (6.56) we have,

\[
\Pi_1^2 = \frac{p_1(-2\alpha Z - 2\alpha p_1 + \alpha p_2 + \alpha b_2 + \alpha t) + f_1(4\alpha p_1 - \alpha p_2 - \alpha b_2 - \alpha t + 4Zt) - 4tf_1^2 + a\alpha^2 - 2a\alpha}{\alpha(2t - \alpha)}
\]  

(6.57)

To maximize this value we find the first order condition.

\[
\frac{d\Pi_1^2}{df_1^*} = \frac{4\alpha p_1 - \alpha p_2 - \alpha b_2 - \alpha t + 4Zt - 8tf_1^*}{\alpha(2t - \alpha)} = 0
\]

As \( \frac{d^2\Pi_1^2}{df_1^2} = -\frac{8t}{\alpha(2t - \alpha)} < 0 \), the second order condition for maximization is also satisfied.

Solving for the F.O.C. we find \( f_1^* = \frac{4\alpha p_1 - \alpha p_2 - \alpha b_2 - \alpha t + 4Zt}{8t} \). Next we will solve the first stage pricing problems of both the firms.

6.5.3 Stage 1: Pricing Game

In this stage both firms will try to maximize their profits by setting their prices. We assume that both the firms will be able to anticipate its competitor’s coupon face
value in the following stages of the game. Plugging the values of $f_1^*$ and $f_2^*$ into the
equations (6.55) and (6.57) gives us following two first stage profit function that we need
to maximize simultaneously by setting the values of $p_1$ and $p_2$.

$$\Pi_1^* = \frac{p_1(16t\alpha(\alpha + b_2) - 8\alpha^2(t + b_2)) + p_2(-8t\alpha Z + 2\alpha^2(b_2 + t)) + 8\alpha p_1 p_2(2t - \alpha)}{16t\alpha(2t - \alpha)}$$

$$+ \frac{-16\alpha p_1^2(2t - \alpha) + \alpha^2 p_2^2 - 16at(2t - \alpha) - 8t\alpha Z(t + b_2) + \alpha^2b_2(2t + b_2) + t^2(\alpha^2 + 16Z^2)}{16t\alpha(2t - \alpha)}$$

(6.58)

$$\Pi_2^* = \frac{p_2(-4tb_2 - 3t\alpha + 4t^2 + \alpha b_2 + 4tZ) + p_2^2(\alpha - 6t) + 4p_1 p_2(2t - \alpha) + 2tb_2^2}{4t(2t - \alpha)}$$

(6.59)

In order to solve the problems simultaneously, we find the first order conditions for both
and solve them simultaneously.

F.O.C.

$$\frac{d\Pi_1^*}{dp_1} = \frac{t\alpha^2 + \alpha(2t - \alpha)(b_2 + p_2) - 4\alpha p_1(2t - \alpha)}{2t\alpha(2t - \alpha)} = 0$$

$$\frac{d\Pi_1^*}{dp_2} = \frac{-4tb_2 - 3t\alpha + 4t^2 + \alpha b_2 + 4tZ + 2p_2(\alpha - 6t) + 4p_1(2t - \alpha)}{4t(2t - \alpha)} = 0$$

As $\frac{d^2\Pi_1^*}{dp_1^2} = -\frac{2}{t} < 0$ and $\frac{d^2\Pi_1^*}{dp_2^2} = -\frac{(6t - \alpha)}{2t(2t - \alpha)} < 0$, the second order conditions for
maximization are also satisfied. Solving both first order conditions simultaneously gives

$$p_1^* = \frac{16t^2 + 8tb_2 + 4tZ - 5t\alpha - \alpha b_2}{4(10t - \alpha)}$$

and

$$p_2^* = \frac{6t^2 - 2tb_2 + 4tZ - 4t\alpha}{10t - \alpha}.$$ Additionally, $f_1^* = \frac{Z}{2}$
and \( f_2^* = \frac{6t^2 - 12tb_2 + 4tZ - 4t\alpha + ab_2}{2(10t - \alpha)} \). We also have \( x_1^* = \frac{Z}{2\alpha} \),

\[
x_2^* = \frac{16t^2 + 8tb_2 + 4tZ - 5t\alpha - ab_2}{8t(10t - \alpha)}, \quad x_3^* = \frac{16t^2 + 8tb_2 - 16tZ - 5t\alpha - ab_2 + 2Z\alpha}{4(10t - \alpha)(2t - \alpha)},
\]

\[
x_4^* = \frac{28t^2 - 16tb_2 + 12tZ - 13t\alpha + ab_2}{8t(10t - \alpha)} \quad \text{and} \quad x_5^* = \frac{28t^2 - 16tb_2 + 8tZ - 13t\alpha + ab_2 + 2Z\alpha}{4(10t - \alpha)(2t - \alpha)}. \quad \text{The}
\]

market shares are \( k_1^* = \frac{16t^2 + 8tb_2 - 16tZ - 5t\alpha - ab_2 + 2Z\alpha}{2(10t - \alpha)(2t - \alpha)} \) and

\[
k_2^* = \frac{24t^2 - 8tb_2 + 16tZ - 19t\alpha + ab_2 - 2Z\alpha + 2\alpha^2}{2(10t - \alpha)(2t - \alpha)}.
\]

The profits for the two firms are.

\[
\Pi_1^* = -\frac{256\alpha^2 b_2 Z + 64\alpha^3 b_2 Z - 4\alpha^3 b_2 Z + 64\alpha^3 b_2^2 + 256\alpha^3 b_2 - 112\alpha^2 t^2 b_2 - 512\alpha t^3 Z}{8\alpha(10t - \alpha)^3(2t - \alpha)} \quad \text{and} \quad \Pi_2^* = \frac{224\alpha^2 t^2 Z - 16\alpha^2 t b_2^2 + 10\alpha^2 t b_2 - 304\alpha^2 t Z^2 + \alpha^2 b_2^2 - 20\alpha^2 tZ + 24\alpha^2 t^2 + 256\alpha t^4}{8\alpha(10t - \alpha)^3(2t - \alpha)},
\]

\[
= \frac{-160\alpha^2 t^3 + 25\alpha^3 t^2 + 800\alpha^3 t^2 + 1120\alpha^2 t^2 - 176\alpha^2 t + 8\alpha^4 - 1600\alpha t^3}{8\alpha(10t - \alpha)^3(2t - \alpha)}.
\]

\[
\Pi_1^* = -\frac{-8\alpha^2 t + 16\alpha^2 tZ - 8\alpha tZ^2 + 8ab_2 tZ + \alpha^2 b_2^2 + 72\alpha^2 t^2 + 48t^2 Z^2 - 8\alpha^2 b_2 t - 120\alpha t^2 Z}{2(10t - \alpha)^2(2t - \alpha)} \quad \text{and} \quad \Pi_2^* = \frac{-162\alpha^3 + 144t^3 Z + 108t^4 - 22ab_2^2 t + 60ab_2 t^2 - 48b_2 t^2 Z + 112b_2 t^3 - 72b_2 t^3}{2(10t - \alpha)^2(2t - \alpha)}.
\]

The following proposition summarizes this model.

**PROPOSITION 11:** Under Assumption 1, the game between the two firms has a unique Subgame Perfect Equilibrium characterized as,

\[
\{p_1^*, p_2^*; f_1^*, f_2^*; k_1^*, k_2^*; \Pi_1^*, \Pi_2^*\} =
\]
6.6 Case 6: Conventional Paper Coupon User Firm vs. Conventional Paper Coupon User Firm

In this section we model the competition between two conventional paper coupon user firms. The game between these two firms is modeled as a two-stage game similar to Case 4 of Table 6. In the first stage both firms determine their prices. In the second stage both firms decide on the face value of their coupons simultaneously and distribute them. We illustrate all the possible types of marginal consumers with the help of Figure 18.

In the figure, the relative locations of marginal consumer types are given by

\[ x_i \in [0, \frac{1}{2}] \] (i = 1, 2, 3, 4, 5, 6) defined as the distance of marginal consumer type i to Firm 1.
With five marginal consumer types we will have up to seven different regions (I, II, III, IV, V, VI, VII). Before we explain how this works we need to state what each of the five marginal consumer types means. Table 17 gives detailed descriptions of these marginal consumers.

The following equations show indifference of each marginal consumer type mathematically.

Table 17 Marginal Consumer Types and Descriptions for Case 6

<table>
<thead>
<tr>
<th>Marginal Consumer Type</th>
<th>Distance from Firm 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1$</td>
<td>Indifferent between using or not using the Firm 1’s paper coupon.</td>
</tr>
<tr>
<td>2</td>
<td>$x_2$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 2’s coupon but not Firm 1’s.</td>
</tr>
<tr>
<td>3</td>
<td>$x_3$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has both the Firm 2’s and Firm 1’s coupons.</td>
</tr>
<tr>
<td>4</td>
<td>$x_4$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has none of the coupons.</td>
</tr>
<tr>
<td>5</td>
<td>$x_5$</td>
<td>Indifferent between buying from Firm 1 or Firm 2 given she has the Firm 1’s coupon but not Firm 2’s.</td>
</tr>
<tr>
<td>6</td>
<td>$x_6$</td>
<td>Indifferent between using or not using the Firm 2’s paper coupon.</td>
</tr>
</tbody>
</table>
\[ f_i = Z - x_i \alpha \quad (6.60) \]

\[ p_1 + x_2 t = p_2 + \left( \frac{1}{2} - x_2 \right) t - f_2 + \left( Z - \alpha \left( \frac{1}{2} - x_2 \right) \right) \quad (6.61) \]

\[ p_1 + x_3 t - f_1 + (Z - x_3 \alpha) = p_2 + \left( \frac{1}{2} - x_3 \right) t - f_2 + (Z - \alpha \left( \frac{1}{2} - x_3 \right) \alpha) \quad (6.62) \]

\[ p_1 + x_4 t = p_2 + \left( \frac{1}{2} - x_4 \right) t \quad (6.63) \]

\[ p_1 + x_5 t - f_1 + (Z - x_5 \alpha) = p_2 + \left( \frac{1}{2} - x_5 \right) t \quad (6.64) \]

\[ f_2 = Z - \left( \frac{1}{2} - x_6 \right) \alpha \quad (6.65) \]

Solving for \( x \)'s we have,

\[ x_1 = \frac{Z - f_1}{\alpha} \quad (6.66) \]

\[ x_2 = \frac{p_2 - p_1 - f_2 + Z + \frac{t - \alpha}{2}}{2t - \alpha} \quad (6.67) \]

\[ x_3 = \frac{p_2 - p_1 + f_1 - f_2 + \frac{t - \alpha}{2}}{2(t - \alpha)} \quad (6.68) \]

\[ x_4 = \frac{p_2 - p_1 + \frac{t}{2}}{2t} \quad (6.69) \]

\[ x_5 = \frac{p_2 - p_1 + f_1 - Z + \frac{t}{2}}{2t - \alpha} \quad (6.70) \]

\[ x_6 = \frac{1}{2} - \frac{Z - f_2}{\alpha} \quad (6.71) \]
Having stated the descriptions and locations of marginal consumer types, we can explain each region with the help of Table 18.

Table 18 Consumer Behavior in Different Regions for Case 5

<table>
<thead>
<tr>
<th>Region</th>
<th>Consumer behavior in the region</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Buys from Firm 1 (does not use Firm 1’s coupon even if receives).</td>
</tr>
<tr>
<td>II</td>
<td>Buys from Firm 1 and uses Firm 1’s coupon if receives.</td>
</tr>
<tr>
<td>III</td>
<td>Buys from Firm 2 if receives only Firm 2’s coupon, otherwise buys from Firm 1 (uses the coupons).</td>
</tr>
<tr>
<td>IV</td>
<td>Buys from Firm 2 if receives Firm 2’s coupon, otherwise buys from Firm 1 (uses the coupons).</td>
</tr>
<tr>
<td>V</td>
<td>Buys from Firm 1 if receives only Firm 1’s coupon, otherwise buys from Firm 2 (uses the coupons).</td>
</tr>
<tr>
<td>VI</td>
<td>Buys from Firm 2 (uses the coupon).</td>
</tr>
<tr>
<td>VII</td>
<td>Buys from Firm 2 (does not use Firm 2’s coupon even if receives).</td>
</tr>
</tbody>
</table>

Now, we can calculate each firm’s payoffs in each of the seven regions. As the firms are symmetric and do not have ability to target coupons they both mass distribute their coupons and all consumers receive both firms’ coupons. It is clear that regions III and V disappear due to this assumption. But we show these regions and corresponding marginal consumer locations in Table 17, Table 18 and Figure 18 as we would like to demonstrate all possible marginal consumer types in a very general setting. In the following sections, the optimization problems do not worry about these regions and marginal consumer types. In the next section we analyze the second stage couponing game, called “Paper Couponing Game,” for the two conventional paper coupon user firms.

6.6.1 Stage 2: Paper Couponing Game

In this stage we make some simplifying assumptions that we made in Case 3 and assume that the distribution intensity ($\lambda$) of both firms’ mass media distributed paper coupon is one. That is, paper coupons will reach all the consumers that are interested in
using them. Therefore at this stage firms are concerned only with the face value of their coupons. The profits of Firm 1 and Firm 2 at this stage, denoted by $\Pi_1$ and $\Pi_2$, can be found as,

$$\Pi_1 = 2p_1x_i + 2(p_i - f_1)(x_3 - x_i) - a. \quad (6.72)$$

$$\Pi_2 = 2p_2\bigg(\frac{1}{2}x_6\bigg) + 2(p_2 - f_2)(x_6 - x_i) - a. \quad (6.73)$$

Substituting the values of corresponding $x$’s from equations (6.66) to (6.71) into equations (6.72) and (6.73) and taking the first order derivatives gives the following first order conditions:

F.O.C.

$$\frac{d\Pi_1}{df_i} = \frac{4\alpha p_1 - \alpha t + 4\alpha f_1 + 2\alpha f_2 - 2\alpha p_2 + \alpha^2 + 4Zt - 4\alpha Z - 8tf_1}{2\alpha(t - \alpha)} = 0$$

$$\frac{d\Pi_2}{df_2} = \frac{4\alpha p_2 - \alpha t + 4\alpha f_2 + 2\alpha f_1 - 2\alpha p_1 + \alpha^2 + 4Zt - 4\alpha Z - 8tf_2}{2\alpha(t - \alpha)} = 0$$

As $\frac{d^2\Pi_1}{df_1^2} = \frac{d^2\Pi_2}{df_2^2} = -\frac{2(2t - \alpha)}{\alpha(t - \alpha)} < 0$ the second order conditions for maximization are also satisfied. Solving for the two first order conditions simultaneously we find

$$f_{1*} = \frac{16\alpha tp_1 - 8\alpha p_2 - 10\alpha^2 p_1 + 8\alpha^2 p_2 - 20\alpha tZ + 16Zt^2 + 4\alpha^2 Z - 4\alpha t^2 + 5\alpha^2 t - \alpha^3}{2(3\alpha^2 - 16\alpha t + 16t^2)}$$

and

$$f_{2*} = \frac{16\alpha tp_2 - 8\alpha p_1 - 10\alpha^2 p_2 + 8\alpha^2 p_1 - 20\alpha tZ + 16Zt^2 + 4\alpha^2 Z - 4\alpha t^2 + 5\alpha^2 t - \alpha^3}{2(3\alpha^2 - 16\alpha t + 16t^2)}$$

Next we will solve the first stage pricing problems of both the firms.
6.6.2 Stage 1: Pricing Game

In this stage both firms will try to maximize their profits by setting their prices. We assume that both the firms will be able to anticipate its competitor’s coupon face value in the second stage of the game. Plugging the values of \( f_1^* \) and \( f_2^* \) into the equations (6.72) and (6.73) and taking the first order derivatives with respect to prices gives us the following two F.O.C.’s:

\[
\frac{d\Pi_1}{dp_1} = \frac{2(448\alpha^2 p_1 - 256\alpha^3 p_1 + 46\alpha^3 p_1 - 256\alpha^2 t p_1 - 256\alpha t^2 p_2 + 128\alpha^3 p_2 - 32\alpha^3 p_2 + 160\alpha^2 p_2)}{(3\alpha^2 - 16\alpha t + 16t^2)^2} + \frac{2(112\alpha^2 t^2 - 36\alpha^3 - 7\alpha^3 Z + 48\alpha^2 tZ - 144\alpha^2 t^3 - 96\alpha^2 Z + 64t^4 + 64 t^3 Z + 4\alpha^4)}{(3\alpha^2 - 16\alpha t + 16t^2)^2} = 0
\]

\[
\frac{d\Pi_2}{dp_2} = \frac{2(448\alpha^2 p_2 - 256\alpha^3 p_2 + 46\alpha^3 p_2 - 256\alpha^2 t p_2 - 256\alpha t^2 p_1 + 128\alpha^3 p_1 - 32\alpha^3 p_1 + 160\alpha^2 p_1)}{(3\alpha^2 - 16\alpha t + 16t^2)^2} + \frac{2(112\alpha^2 t^2 - 36\alpha^3 - 7\alpha^3 Z + 48\alpha^2 tZ - 144\alpha^2 t^3 - 96\alpha^2 Z + 64t^4 + 64 t^3 Z + 4\alpha^4)}{(3\alpha^2 - 16\alpha t + 16t^2)^2} = 0
\]

As \( \frac{d^2\Pi_1}{dp_1^2} = \frac{d^2\Pi_2}{dp_2^2} = \frac{128t^2 (7\alpha - 8t) + 2\alpha^2 (46\alpha - 256t)}{(3\alpha^2 - 16\alpha t + 16t^2)^2} < 0 \) (easily verified with \( t > \alpha \)), the second order conditions for maximization are also satisfied. Solving both first order conditions simultaneously gives,

\[
p_1^* = p_2^* = \frac{16t^3 + 16t^2 Z - 32\alpha t^2 - 20\alpha tZ + 20\alpha^2 t - 4\alpha^3 + 7\alpha^2 Z}{2(16t^2 - 20\alpha t + 7\alpha^2)}. \]

Additionally, we have \( f_1^* = f_2^* = \frac{16t^2 Z - 20\alpha tZ + 7\alpha^2 Z + \alpha^2 t - \alpha^3}{2(16t^2 - 20\alpha t + 7\alpha^2)}. \) We also have

\[
x_1^* = \frac{16t^2 Z - 20\alpha tZ + 7\alpha^2 Z - \alpha^2 t + \alpha^3}{2\alpha(16t^2 - 20\alpha t + 7\alpha^2)};
x_3^* = \frac{1}{4}, \quad x_4^* = \frac{1}{4} \text{ and}
\]
\[ x_b^* = \frac{16\alpha^2 - 19\alpha^2 t + 6\alpha^3 - 16t^2Z + 20\alpha t Z - 7\alpha^2 t}{2\alpha(16t^2 - 20\alpha t + 7\alpha^2)} \]. The market shares are \( k_1^* = k_2^* = \frac{1}{2} \).

The profits for the two firms are:

\[
\Pi_1^* = \Pi_2^* = \frac{512t^4Z^2 + 2560\alpha\alpha^2t^3 + 98\alpha^4Z^2 + 1120\alpha^2\alpha^4t - 196\alpha\alpha^5}{4\alpha(16t^2 - 20\alpha t + 7\alpha^2)^2} \\
+ \frac{-2496\alpha\alpha^3t^2 - 560\alpha\alpha^3tZ^2 + 1248\alpha^3t^2Z^2 - 1280\alpha\alpha^3Z^2 - 1024\alpha^2t^4}{4\alpha(16t^2 - 20\alpha t + 7\alpha^2)^2} \\
+ \frac{-23\alpha^6 + 197\alpha^5t - 654\alpha^4t^3 + 1056\alpha^3t^3 - 832\alpha^2t^4 + 256\alpha t^5}{4\alpha(16t^2 - 20\alpha t + 7\alpha^2)^2}
\]

The following proposition summarizes this model.

**PROPOSITION 12:** Under Assumption 1, the game between the two firms has a unique Subgame Perfect Equilibrium characterized as,

\[
\left\{ p_1^*, p_2^*; f_1^*, f_2^*; k_1^*, k_2^*; \Pi_1^*, \Pi_2^* \right\} =
\begin{cases}
\frac{16t^4 + 16t^2Z - 32\alpha t^2 - 20\alpha t Z + 20\alpha^2t - 4\alpha^3 + 7\alpha^2Z}{2(16t^2 - 20\alpha t + 7\alpha^2)^2}, & k_1^* \in \left\{ 0, \frac{1}{2} \right\}, \\
\frac{16t^4 + 16t^2Z - 32\alpha t^2 - 20\alpha t Z + 20\alpha^2t - 4\alpha^3 + 7\alpha^2Z}{2(16t^2 - 20\alpha t + 7\alpha^2)^2}, & k_2^* \in \left\{ 0, \frac{1}{2} \right\}, \\
\frac{512t^4Z^2 + 2560\alpha\alpha^2t^3 + 98\alpha^4Z^2 + 1120\alpha^2\alpha^4t - 196\alpha\alpha^5 - 2496\alpha\alpha^3t^2 - 560\alpha\alpha^3tZ^2 + 1248\alpha^3t^2Z^2}{4\alpha(16t^2 - 20\alpha t + 7\alpha^2)^2}, & \Pi_1^* \in \left\{ 0, \frac{1}{2} \right\}, \\
\frac{512t^4Z^2 + 2560\alpha\alpha^2t^3 + 98\alpha^4Z^2 + 1120\alpha^2\alpha^4t - 196\alpha\alpha^5 - 2496\alpha\alpha^3t^2 - 560\alpha\alpha^3tZ^2 + 1248\alpha^3t^2Z^2}{4\alpha(16t^2 - 20\alpha t + 7\alpha^2)^2}, & \Pi_2^* \in \left\{ 0, \frac{1}{2} \right\}.
\end{cases}
\]

### 6.7 Discussion and Analysis of Results

#### 6.7.1 Competitive Internet Couponing Models

We start our analysis with the competition between Internet coupons user firms that are given in Cases 2 and 4 of Table 6. When one firm has the ability to use Internet coupons and the other does not have this ability or choose not to use Internet coupons, the
firm using Internet coupons has a clear advantage for some values of customer profiling
cost parameter, $b_2$. Figure 19 shows the profits of both firms with respect to parameter
$b_2$. Lines 1 and 2 show the two firms’ profits. The term “No Coupons” represents the
profit of the firm that does not use any coupons at all. The term “Internet Coupons”
represents the profit of the firm that uses Internet coupons. Line 3 shows the profits if
none of the firms uses coupons. We call this figure “No Coupons Competition” profit
and it is the profit of Case 1 of Table 6.

It is easily verified that the profits of the Internet coupon user firm is greater than
the no coupon user firm when $b_2 < \frac{t}{2}$. However the bound is even tighter as the Internet

![Figure 19 Profits for Internet vs. No Coupon Competition, Case 2](image)
coupon user firm will choose not to use coupons when $b_2 < \frac{t}{7}$ since its profits will drop below what it would get without targeting coupons.

When both firms use Internet coupons, they both lose as seen in Figure 20. The highest they can make is equal to that of no coupons competition, this happens when $b_2 = \frac{t}{2}$. Actually, at this value of the customer profiling cost parameter, both firms choose not to use coupons and the optimal face value of their coupon becomes zero. Figure 20 exhibits a very important result of our models; both firms’ profits increase as customer profiling cost increases.

If the profiling parameter is zero, the lowest profit will be achieved for the firms at $\frac{3t}{16}$. This means the improvements in customer profiling technology will lead to

![Figure 20 Profits for Internet Coupon Competition, Case 4](image)
intensified competition in which both players stand to lose profits. Our results for Case 4 show that firms will have to keep their prices the same even if they start providing Internet coupons. This is in contrast with the findings of Chapter 5 for a monopolistic firm. We showed there that a monopolist is able to increase its price when it provides Internet coupons. The result here is due to competitive pressures. To summarize, this means consumers will benefit from Internet coupon distribution not coupon providers in a duopoly if both firms have the technology to target coupons.

The competition with Internet coupons can be summarized as a normal form game given in Table 19. The result of this game is prisoners’ dilemma. The competition results in an inefficient outcome for both firms. The following proposition summarizes this result. This result confirms the result found in Shaffer and Zhang (1995) for targeted paper coupons. The Internet does not improve profits but increase the competition.

**PROPOSITION 13:** When both firms have the ability to distribute Internet coupons, the game between the firms is prisoners’ dilemma.

### Table 19 Internet Coupon Competition

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Coupon</td>
<td>Internet Coupon</td>
</tr>
<tr>
<td>No Coupon</td>
<td>$\frac{t}{4}$</td>
</tr>
<tr>
<td>Internet Coupon</td>
<td>$\frac{27t^2 - 18tb_2 + 28b_2^2}{100t}$, $\frac{3t^2 + 4b_2^2}{16t}, \frac{3t^2 + 4b_2^2}{16t}$</td>
</tr>
</tbody>
</table>
6.7.2 Competitive Paper Couponing Models

The analytical results of competitive models for paper coupons cannot be interpreted as easily as the results of Internet couponing models. Even though we could get analytical results in the preceding sections of this chapter, we need to resort to numerical analysis in this section to interpret these results. Specifically we let $t = 5$, $Z = 1$ and $a = 0.05$ in what follows in this section. These values are the same as the values that we used in Chapter 4 to analyze a monopolist’s decision to use paper coupons to allow comparisons. Figure 21 shows the results of two models: paper vs. no coupons and paper vs. paper coupons, namely Cases 3 and 6 of Table 6. It is noted in the figure that the values for $\alpha$ are constrained between 2.084 and 5. Within these limits the constraints of the corresponding models given in Cases 3 and 6 are satisfied, i.e. the relative ordering of $x$’s is preserved and $\alpha < t$.

One of the important results that emerge in the figure is the relationship between the profits of both firms and $\alpha$. As $\alpha$ increases the profits for all players go down (lines 2, 3 and 4 are below line 1 for all feasible values of parameter $\alpha$). This is because as $\alpha$ increases more consumers use coupons and this stiffens the competition between the firms. This is the exact opposite of what we observed for a monopolist in Chapter 4. A monopolistic firm’s profits would increase as $\alpha$ increases.

A more important result is that the profits for both players are lower than what it would be if they did not use coupons at all. Even if only one of the players distributes coupons, both the firms will have lower profits. Again, this is the opposite of the
Figure 21 Profits for Paper Coupon Competition Models, Cases 3 and 6

monopolistic case where the coupon provision was profitable at least under certain conditions. Here, all feasible values of $\alpha$ result in lower profits for both. If only one firm distributes coupons, there is a threshold value $\alpha^*$ below which the coupon providing firm’s profits fall below its competitor’s profits. Therefore for all values of $\alpha < \alpha^*$ no firms will give away coupons. One incentive for the coupon providing firm is that its market share will be greater than its competitor if it is the sole provider of coupons.

When both firms distribute coupons their profits will be even lower than any other alternative. To find out what outcome is likely in a normal form game between two paper coupon providers we formulate the game in Table 20. It is clear from the above discussion that, for our given example, the outcome (no coupon, no coupon) is the dominant strategy pair and the unique Nash equilibrium as both firms will choose not to
provide coupons if their sole motive is profit maximization and this is what we assume in this work. Therefore, both firms choose not to provide coupons in equilibrium, at least for this numerical example. Although we failed generalize this result to all values of parameters, as the analytical solutions do not grant a straightforward solution, for a large number of numerical examples the result holds.

To summarize, our results indicate that firms will not increase their profits with the distribution of paper coupons in our limited setting. However, firms can gain competitive advantage and increase their market shares with the use of paper coupons.

Table 20 Paper Coupon Competition

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Coupon</td>
</tr>
<tr>
<td></td>
<td>( \frac{t}{4}, \frac{t}{4} )</td>
</tr>
<tr>
<td>No Coupon</td>
<td>(-176aZt^2 + 24a^2Zt + 144at^2 - 120at^2 )</td>
</tr>
<tr>
<td>Paper Coupon</td>
<td>(-25at^2 - 57baat^2 + 480aat^2 + 112aat^2 + 8aaat )</td>
</tr>
<tr>
<td></td>
<td>( (\theta - 2a + Z)(12t^2 - 11at + 8Z - 2aZ + 2a^2) )</td>
</tr>
</tbody>
</table>

6.7.3 Competitive Internet vs. Paper Couponing Models

In this section we compare the profits of paper and Internet coupon distributing firms if individual firms use different couponing techniques. Figure 22 and Figure 23
show the numerical results. Once again, the analytical results are cumbersome to work with, as shown by the normal form game for this case given in Table 21, so we have to resort to numerical analysis. In this section we let \( t = 5, Z = 1 \) and \( a = 0.05 \). The value of \( \alpha \) is set to 3 in Figure 22 to analyze the effect of \( b_2 \) on profits. In this case the constraints (the relative ordering of \( x \)'s) of the corresponding model given in case 5 are satisfied with \( b_2 \in [0, 1.589] \). As seen in Figure 22, for all feasible values of \( b_2 \), the firms using Internet coupons make more profit than the paper coupon user firm. Our analysis also indicates a higher market share for the Internet firm.

![Figure 22 Internet vs. Paper Coupon Competition, Case 5: Effect of \( b_2 \)](image-url)
Table 21 Internet vs. Paper Coupon Competition

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>No Coupon</th>
<th>Internet Coupon</th>
<th>Paper Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Coupon</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>2</td>
<td>3</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Firm 2</th>
<th>No Coupon</th>
<th>Internet Coupon</th>
<th>Paper Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32</td>
<td>2</td>
<td>3</td>
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<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\[27r^2 - 8th + 28b^y = \frac{(2 + h)y}{100t}, \quad \frac{(2t + h)y}{25t}\]

\[-192azr^z + 128axr^x - 20azr^z + 288z^r, \quad \frac{8(a + t - \alpha)(2t - \alpha)}{2(a + t - \alpha)}\]

\[-176azr^z + 24ar^z + 144axr^x - 120z^r, \quad \frac{8(a + t - \alpha)z + 2(a + t - \alpha)}{2(a + t - \alpha)}\]

\[25x^r - 576ax^r + 480ax^2r^x - 112axt^r + 8ax^t, \quad \frac{8(a + t - \alpha)(2t - \alpha)}{2(a + t - \alpha)}\]
The Internet firm’s profits decrease as $b_2$ increases whereas the paper coupon firm increases its profits when $b_2$ increases. Considering that $b_2$ is the customer profiling cost this makes perfect sense. Higher the profiling cost harder to target coupons. Therefore the Internet coupon firm will target less number of consumers when it faces with higher profiling costs.

Next we set the value of $b_2$ to 1 in Figure 23 to analyze the effect of $\alpha$ on profits. In this case the constraints (the relative ordering of $x$’s and $\alpha < t$) of the corresponding model given in case 5 are satisfied with $\alpha \in [2.394, 3.834]$.

Figure 23 shows that both firms’ profits fall with higher values of $\alpha$. This is because as $\alpha$ increases more consumers use paper coupons and this stiffens the competition between the firms. The Internet coupon user firm’s profits fall faster than its competitor. Because the Internet coupon firm has to decrease its price to lure the people who are willing to otherwise use paper coupons. This leads to lower profits for the paper coupon firm as well since it should in turn reduce its price to meet the threat presented by the lower prices of the Internet coupon firm.

The most important result that emerges from Figure 22 and Figure 23 is that the profits of both firms are lower than the profits with no coupons (line 3). Even the Internet coupon user firm with its customer profiling technology will not be able to improve its profits over its paper coupon using competitor. No matter what the cost of customer profiling is, the Internet firm cannot improve its profits if its competitor uses paper coupons. To test this we set the value of $b_2$ to zero to see what happens to the profits. Even when this happens we get the same result.
Figure 23 Internet vs. Paper Coupon Competition, Case 5: Effect of $\alpha$
CHAPTER 7
CONCLUSIONS, LIMITATIONS AND FUTURE WORK

This dissertation is devoted to developing an analytical framework to analyze one of least researched, yet important, elements of Internet Marketing mix: Internet coupons. A set of monopolistic and duopolistic models is developed to investigate how Internet coupons might and should be used differently compared to more traditional ways of coupon distribution.

The technologies that empower Internet coupons as one-to-one marketing tools are reviewed in detail in Chapter 2. We discuss different Internet couponing business models, their mechanisms for security and information collection as well as pros and cons of each Internet couponing alternative in the same chapter. After the review of Internet technology for coupons, we gave a brief review of extant literature in Chapter 4 and then we return to our main objective and model the effects of these new developments in Chapters 4, 5 and 6.

We start our models in Chapter 4 and model the conventional mass media distributed paper coupons by extending the unit circle model of Salop (1979). We formally introduce the notion of “disutility of using coupons” in the model in this chapter. We show that by using a conventional paper coupon, a monopolist improves its profits (under very general conditions) and increases its market coverage (i.e. sells to more consumers). Our results indicate that the monopolist does not have to change its price structure compared to the no coupon case when it gives out coupons. The same price prevails as the optimal price even with the use of coupons. We argue that this
situation is unexpected and contrary to the conventional wisdom that would suggest that the firm should increase its price in the presence of coupons. The model indicates that the firm is able to price discriminate between coupon users and nonusers without a need to increase its price. Our solutions further establish that the face value of the coupon depends only on the disutility of using coupons for those who have the highest preference for the product. This is because we assume a uniform distribution for the disutility of using paper coupons. It seems that when the distribution is uniform no other parameter of the model affects the coupon face value. Other distributions may be tested. We reserve this as a future research endeavor and a possible extension to our model for the paper coupons.

Another important result of the model of Chapter 4 is that the cost of coupon distribution does not affect the existence of couponing in the monopolistic setting. It does, however, effect the fraction of consumers who get a coupon and the effectiveness of couponing in terms of additional profit it generates. We also analyze the effect of the decrease in the disutility of using coupons parameter, $\alpha$. We demonstrate that the profit of the monopolist increases with increased $\alpha$. As we mention in Chapter 4, $\alpha$ can be thought of as a metric that measures the level of dispersion among consumers in terms of their disutility of using paper coupons. As $\alpha$ increases, the spread of distribution becomes greater. Alternatively, we can think of $\alpha$ as the difference between the fit costs of coupon users and nonusers. To reflect the fact that the coupon users are more price-sensitive, we let their fit cost to be lower than that of nonusers when they receive coupons. In both interpretations, it becomes possible for the firm to discriminate between consumers as
two groups (coupon users and nonusers) become more distinct. Hence, the monopolist improves its profits with higher values of $\alpha$.

In Chapter 5 we develop models to analyze different Internet couponing techniques and strategies for a firm in a monopolistic setting. Our models illustrate that the differences of Internet coupons demand a careful consideration when choosing a couponing strategy. Firms should implement the strategy that best suits the technological capabilities of their information systems. For instance, haphazard distribution of coupons over the Internet may not be profitable if using these coupons becomes extremely easy for consumers. We show in Chapter 5 that in order for the firm to still be able to price discriminate, it needs a mechanism to impose a certain level of effort for consumers to use coupons, even for Internet coupons if they are mass distributed with no targeting. Therefore, the disutility of using coupons consumers incur is very important to use of Internet for mass distribution of coupons.

Next, we explore the targeted distribution of Internet coupons as an alternative to mass distribution. First, we look at the perfect information case and the results of the basic model were the same as those of perfect (first-degree) price discrimination. Each individual consumer is charged her true reservation utility for the product. We argue that even though the end result is the same with any given perfect dynamic pricing mechanism, the two alternatives arouse different perceptions among consumers. Anecdotal evidence shows that the situation is rather different with coupons and the use of coupons as price discrimination tool does not seem to have the same adverse effect that dynamic pricing has on consumers. We hypothesize that one possible reason why this occurs may be the fact that the price is pre-announced and known by all consumers
when a coupon is used as a price discrimination tool. All consumers are aware of this price but not necessarily the coupons others get.

After looking at rather trivial perfect information case, we turn our attention to targeting with imperfect information. The first model in that section deals with fixed face value targeted Internet coupons and the second model with the changing face value Internet coupons. In both these models we found that the optimal solution requires the monopolist to increase its price. When targeting Internet coupons, a monopolistic firm should increase the price for consumers with high valuation for the product; therefore, the consumers without a coupon must swallow a higher price. We show that this price is higher than the price that would be charged if no coupons were targeted. This is in contrast to what we found for the conventional paper coupons where the monopolist did not have to increase its price to improve its profits with paper coupon promotions.

The results given in Chapter 5 also attest that the improvements in customer profiling technologies that lead to cost reductions will make the monopolistic firm charge more for its product, which means that if the profiling technology becomes cheaper, the firm will try to target more consumers, i.e. it will be willing to give coupons to more consumers.

As for the two Internet couponing techniques we develop in Chapter 5, which one is better for a monopolist? Our answer to this question is mixed. If the targeting accuracy cost, which represents the accuracy of estimating individual consumer’s fit cost, is comparatively high and the accuracy levels are not sufficient, it is not preferable to use the changing face value Internet coupons. The monopolist is better off using the fixed face value coupons. Furthermore, as the customer profiling cost increases, ceteris
paribus, the changing face value Internet coupons become less profitable than the fixed face value coupons. In some extreme cases for certain values of targeting accuracy cost, no accuracy level can justify the implementation of changing face value Internet coupons. This might explain why the technologies we discuss in Chapter 2 to deliver customized coupons are not widely used in practice despite the existence of techniques to provide the changing face value Internet coupons. The reason for this is the high targeting costs and/or the lack of high accuracy levels.

The results discussed so far are for monopolistic cases. What happens when we have competition? Do the same results prevail? Do firms increase their prices? How about their profits? Chapter 6 attempts to answer these questions where the competition between two firms using different couponing alternatives is modeled as a duopoly. We find that some of the results for the monopolistic case change in a competitive environment.

First of all, the competition between two firms that both have the ability to use Internet coupons leads to prisoners’ dilemma. The highest profit these two firms can make is equal to that of no coupons competition and this happens at the highest possible customer profiling cost figure. Above this figure both firms prefer not to use coupons and the optimal face value of their coupon is zero. As a matter of fact, both firms’ profits increase in customer profiling cost. This means the developments in customer profiling technology that reduces the profiling cost will lead to intensified competition in which both players stand to lose profits. For instance, if the profiling cost becomes zero, both firms make the lowest profit possible. These lower profits are due to the firms’ inability to increase their prices when they provide Internet coupons. Our results in Chapter 6
reveal that the prices that are optimal for the no coupons case emerge as the equilibrium prices for the Internet coupons competition. This is in contrast with our solutions for a monopolistic Internet coupon user firm where the firm increases its price when it uses Internet coupons.

As a result, consumers will benefit from Internet coupon distribution not coupon providers if both firms have the technology to target coupons. Having said this, the situation is little different when one of the firms does not have the ability to use Internet coupons. In this case, the firm using Internet coupons can garner more proceeds than its competitor for some values of customer profiling cost parameter.

The results for competitive models of conventional paper coupons are not easy to interpret. However, one result is very clear. The relationship between the profits of both the firms and $\alpha$. As $\alpha$ goes up the profits for both the firms go down. This is again because as $\alpha$ increases more consumers use coupons and this stiffens the competition between the firms. This is the exact opposite of what we have for a monopolist where the profits increase as $\alpha$ increases. In a competitive setting the fit cost parameter $t$ also represents the level of brand loyalty for competing firms (Shaffer and Zhang 1995). Therefore, as $\alpha$ increases, the level of brand loyalty each firm enjoys decrease. Hence, consumers become more willing to switch brands. This hardens the competition and lowers the profits for both the firms.

Even though it is hard to get full understanding of analytical results of the competitive cases for paper coupons, we give some numerical examples in Chapter 6. For these examples, the profits for both players are lower than what it would be if they did not use coupons at all, even when only one firm distributes paper coupons. In this
last case the profits are lower than no coupon competition case for both the firms. For our given example, the outcome (no coupon, no coupon) is a dominant strategy pair and the unique Nash equilibrium for paper couponing. We can show the same for a large number of examples; however, we cannot generalize this result to all feasible values of parameters.

Finally, we look at the competition between conventional paper and Internet coupon distributing firms in Chapter 6. Once again, the analytical results do not allow easy interpretations. Our numerical analysis indicates that the Internet firm’s profits decrease as customer profiling cost increases whereas the paper coupon firm’s profits increase when this happens. Additionally, we see that both firms’ profits fall as $\alpha$ goes up. For most values of parameters the Internet coupon user firm makes more profits than its counterpart. However, once again, both firms’ profits are lower than the profits with no coupons competition. Even the Internet coupon user firm with its customer profiling technology will not be able to improve its profits over the no coupon competition case no matter what the cost of customer profiling is including zero.

Our results are rather limited as our models are one-time purchase models and we assume that the market size remains the same when coupons are provided. If the market size is allowed to grow when coupons are provided, the results may be different. In a repeat purchase setting these results for the competitive cases may change and firms may benefit from repeat purchases in the long run. If we allow for repeat purchases, the benefits of customer profiling might also be even greater as firms are able to gather more accurate information in the long term. Additionally, firms with Internet technology can use the same information to target consumers multiple times, which will result in cost
savings. Considering repeat purchases is a possible future extension that we plan to pursue.

Another limitation of our models is that we assume symmetry between the two competing firms. It is assumed in our models that the two firms are identical. In an asymmetric duopoly we may get more insights and explore the differences between levels of firms’ ability to use different couponing techniques. This is a logical extension to our models.

One possible extension to this study is to consider the case of multiple products where the effects of substitution and complementarity among a firm’s products are analyzed. Another possible extension is to consider distributions other than uniform distribution for the spread of consumers on the unit circle. We relegate the task of tackling these extensions to the future.
APPENDIX A
SOLUTIONS

A.1 Solutions for Chapter 4

The solution to conventional paper coupon firm’s problem is given below.

\[
Max_{p, f, t} \left[ \Pi_{\text{Total}} \right] = 2 \left[ \left( \frac{rp - p^2}{t} \right) - \frac{\lambda f r}{t} + \frac{\lambda f p}{t} + \frac{\lambda f Z}{t} - \frac{\lambda f^2}{t} \right] \\
+ \frac{\lambda}{t - \alpha} \left[ pf - f^2 - Zp + Zf + \frac{ar p}{t} - \frac{ar f}{t} - \frac{\alpha p^2}{t} + \frac{\alpha pf}{t} \right] - a\lambda^2
\]

Subject To

\[
\begin{align*}
0 & \leq \lambda \leq 1 \\
0 & \leq p \leq r \\
0 & \leq f \leq p \\
0 & \leq \frac{r - p}{t} \leq \frac{1}{2} \\
0 & \leq \frac{f}{t} \leq 1 - \frac{r - p}{t} \\
0 & \leq \frac{Z - f}{\alpha} \leq \frac{r - p}{t}
\end{align*}
\]

First, we calculate the First Order Conditions (F.O.C.).

\[
\frac{d\Pi_{\text{Total}}}{dp} = 0 = 2 \left[ \left( \frac{r - 2p^*}{t} \right) + \frac{\lambda^* f^*}{t} \right] + \frac{\lambda^*}{t - \alpha} \left[ f^* - Z + \frac{ar p^*}{t} - \frac{2\alpha p^*}{t} + \frac{\alpha f^*}{t} \right]
\]

From this, after some algebra, we have

\[
(r - 2p^*) = \frac{t\lambda^*(Z - 2f^*)}{t - \alpha(1 - \lambda^*)}
\]
\[
\frac{d\Pi_{\text{Total}}}{df} = 0 = 2 \left[ -\frac{\lambda^* r}{t} + \frac{\lambda^* p}{t} + \frac{\lambda^* Z}{\alpha} - \frac{2\lambda^* f^*}{\alpha} \right] + \frac{\lambda^*}{t-\alpha} \left[ p^* - 2f^* + Z - \frac{ar}{t} + \frac{\alpha p^*}{t} \right]
\]

\[
= 2 \left[ -\frac{\lambda^* (r-2p^*)}{t-\alpha} + \frac{\lambda^* t(Z-2f^*)}{\alpha(t-\alpha)} \right].
\]

Hence, we have \( p^* = \frac{r}{2} - \frac{t(Z-2f^*)}{2\alpha} \).

\[
\frac{d\Pi_{\text{Total}}}{d\lambda} = 0 = 2 \left[ -\frac{f^* r}{t} + \frac{f^* p^*}{t} + \frac{f^* Z}{\alpha} - \frac{f^* t}{\alpha} \right]
\]

\[
+ \frac{1}{t-\alpha} \left[ p^* f^* - f^*^2 - Zp^* + Zf^* + \frac{\alpha r p^*}{t} - \frac{\alpha f^*}{t} - \frac{\alpha p^*^2}{t} + \frac{\alpha p^* f^*}{t} \right]
\]

\[
= 2 \left[ -\frac{f^* r}{t} + \frac{2f^* p^*}{t} + \frac{f^* Z}{\alpha} - \frac{f^* t}{\alpha} \right]
\]

\[
- \frac{Zp^*}{t-\alpha} + \frac{\alpha p^* (r-p^*)}{t(t-\alpha)} - 2a\lambda^*
\]

In order to check to second order conditions we need to calculate the Hessian matrix and we need to show that the Hessian matrix is at least negative semidefinite. The Hessian for this model is negative definite for all \( \lambda > 0 \). Therefore the second order condition for maximization is satisfied. The Kuhn-Tucker conditions for this model are also checked and they are satisfied.

Second, combining results of \( \frac{d\Pi_{\text{Total}}}{dp} \) and \( \frac{d\Pi_{\text{Total}}}{df} \) we have

\[
\frac{d\Pi_{\text{Total}}}{dp} + \frac{d\Pi_{\text{Total}}}{df} = 0 = \left[ 2 \left[ \left( \frac{r-2p^*}{t} \right) + \frac{\lambda^* f^*}{t} + \frac{\lambda^*}{t-\alpha} \left[ f^* - Z + \frac{ar}{t} - \frac{2\alpha p^*}{t} + \frac{\alpha f^*}{t} \right] \right] \right]
\]

\[
+ 2 \left[ -\frac{\lambda^* r}{t} + \frac{\lambda^* p}{t} + \frac{\lambda^* Z}{\alpha} - 2\lambda^* f^* \right] + \frac{\lambda^*}{t-\alpha} \left[ p^* - 2f^* + Z - \frac{ar}{t} + \frac{\alpha p^*}{t} \right]
\]

After some algebra this is equal to \( 2 \left[ \frac{(1-\lambda^*)(r-2p^*)}{t} + \frac{\lambda^*(Z-2f^*)}{\alpha} \right] \).
At some point in \( \frac{d\Pi_{\text{Total}}}{df} \) we have \((r-2p^*)=\frac{t(Z-2f^*)}{\alpha}\). Therefore, we have

\[
2 \left[ \frac{(1-\lambda^*)}{t} \frac{t(Z-2f^*)}{\alpha} + \frac{\lambda^*(Z-2f^*)}{\alpha} \right] = 2 \left[ \frac{(Z-2f^*)}{\alpha} \right].
\]

Thus, we have \(f^* = \frac{Z}{2}\). Given this, we can find the price using the result of \( \frac{d\Pi_{\text{Total}}}{dp} \) or \( \frac{d\Pi_{\text{Total}}}{df} \) as

\[
p^* = \frac{r}{2} - \frac{t(Z-2f^*)}{2\alpha} = \frac{r}{2}.
\]

Using the result of \( \frac{d\Pi_{\text{Total}}}{d\lambda} \) we can find \(\lambda^*\) as follows:

\[
0 = 2 \left[ -\frac{f^*r}{t-\alpha} + \frac{2f^*p^*}{t-\alpha} + \frac{f^*Zt}{\alpha(t-\alpha)} - \frac{f^{*2}t}{t-\alpha} - \frac{Zp^*}{t-\alpha} + \frac{\alpha p^*(r-p^*)}{t(t-\alpha)} \right] - 2a\lambda^*,
\]

\[
\lambda^* = \frac{1}{a} \left[ -\frac{f^*r}{t-\alpha} + \frac{2f^*p^*}{t-\alpha} + \frac{f^*Zt}{\alpha(t-\alpha)} - \frac{f^{*2}t}{t-\alpha} - \frac{Zp^*}{t-\alpha} + \frac{\alpha p^*(r-p^*)}{t(t-\alpha)} \right].
\]

After some algebra \(\lambda^* = \frac{(ar-Zt)^2}{4a\alpha t(t-\alpha)}\). Market coverage can now be calculated as

\[
d^* = \frac{r-p^*}{t} = \frac{r}{2} = \frac{r}{2t}.
\]

Additionally, we have

\[
x = \frac{f^*-Z+\alpha d^*}{t-\alpha} = \frac{Z-\alpha \frac{r}{2t}}{t-\alpha} = \frac{(ar-Zt)}{2t(t-\alpha)}.
\]

Thus \(\bar{x} = \lambda x = \frac{(ar-Zt)^2}{4a\alpha t(t-\alpha)} \frac{ar-Zt}{2t(t-\alpha)} = \frac{(ar-Zt)^3}{8a\alpha t^2(t-\alpha)^2}\). Hence the market coverage is

\[
k^* = 2x + 2d = 2 \frac{r}{2t} + 2 \frac{(ar-Zt)^3}{8a\alpha t^2(t-\alpha)^2} = \frac{r}{t} + \frac{(ar-Zt)^3}{4a\alpha t^2(t-\alpha)^2}.
\]

We can also find the Coupon Redemption Rate (CRR) as follows. First, we need to find the total fraction of consumers who use coupons, denoted Total Coupon Users (TCU), as
Then, we find the CRR as
\[ \text{CRR} = \frac{TCU}{\lambda} = \frac{(\alpha r - Zt)}{4a\epsilon^2(t-\alpha)^2} \cdot \]

Finally, the total profits of the firm can be calculated by substituting \( p^*, f^* \) and \( \lambda^* \) in the profit formula.

\[ \Pi^*_\text{total} = 2 \left[ \left( \frac{rp^* - p^{*2}}{t} \right) + \lambda^* \left[ \frac{-f^*r}{t} + \frac{f^*p^*}{t} + \frac{f^*Z}{\alpha} - \frac{f^{*2}}{\alpha} \right] \right]
+ \frac{\lambda^*}{t - \alpha} \left[ \frac{p^*f^* - f^{*2}}{t} - Zp^* + Zf^* + \frac{\alpha r p^*}{t} - \frac{\alpha r f^*}{t} - \frac{\alpha p^{*2}}{t} + \frac{\alpha p^* f^*}{t} \right] - a\lambda^{*2}
\]

\[= 2 \left[ \left( \frac{r^2}{2} - \frac{(r^2)}{2} \right) + \frac{(\alpha r - Zt)}{4a\epsilon^2(t-\alpha)} \left[ \frac{r}{2} - \frac{Z}{2} + \frac{Z}{2} + \frac{Z}{\alpha} - \frac{(\frac{Z}{2})^2}{\alpha} \right] \right]
+ \frac{(\alpha r - Zt)}{4a\epsilon^2(t-\alpha)} \left[ \frac{r Z}{2} - \frac{(Z^2)}{2} - \frac{r}{2} + \frac{Z}{2} + \frac{\alpha r}{2} + \frac{\alpha r Z}{2} - \frac{\alpha (\frac{r}{2})^2}{t} + \frac{\alpha r Z}{2} \right]
- a \left[ \frac{(\alpha r - Zt)}{4a\epsilon^2(t-\alpha)} \right]^2\]

After some algebra \( \Pi^*_\text{total} = \frac{r^2}{2t} + \frac{(\alpha r - Zt)^4}{16a\epsilon^2t^2(t-\alpha)^2} \).

A.2 Solutions for Chapter 5

A.2.1 Internet Mass Distribution of Coupons

\[ \max_{p,f,\lambda} \left[ \frac{1}{2} \left[ rp - p^2 - \lambda rf + 2\lambda pf - \lambda f^2 \right] \right] - a_1 - b_1 \lambda \]

Subject To
\[0 \leq \lambda \leq 1
\]
\[0 \leq p \leq r
\]
\[0 \leq f \leq p
\]
\[0 \leq \frac{r - p}{t} \leq \frac{1}{2}
\]
\[0 \leq \frac{f}{t} \leq \frac{1}{2} - \frac{r - p}{t}
\]

F.O.C.

\[
\frac{d\Pi_{\text{Total}}}{dp} = 0 = 2 \left( \frac{r - 2p^* + 2\lambda^* f^*}{t} \right)
\]
\[
\frac{d\Pi_{\text{Total}}}{df} = 0 = 2 \left( -\lambda^* r + 2\lambda^* p^* - 2\lambda^* f^* \right) / t
\]
\[
\frac{d\Pi_{\text{Total}}}{d\lambda} = 0 = 2 \left( -rf^* + 2p^* f^* - f^{*2} \right) / t - b_i
\]

In order to check to second order conditions we need to calculate the Hessian matrix and we need to show that the Hessian matrix is at least negative semidefinite. The Hessian for this model is negative semidefinite. Therefore the second order condition for maximization is satisfied. We also checked the Kuhn-Tucker conditions for the optimal values of the decision variables and they are satisfied.

From \( \frac{d\Pi_{\text{Total}}}{dp} = 0 \) we have \( p^* = \frac{r + 2\lambda^* f^*}{2} \).

From \( \frac{d\Pi_{\text{Total}}}{df} = 0 \) we have \( p^* = \frac{r + 2f^*}{2} \) provided \( \lambda^* \neq 0 \).

So we have \( \frac{r + 2\lambda^* f^*}{2} = \frac{r + 2f^*}{2} \) or \( 2\lambda^* f^* = 2f^* \) that leads to \( (1 - \lambda^*) f^* = 0 \). There are two cases; either \( f^* \) is 0 and \( \lambda^* \) is free or \( \lambda^* \) is 1 and \( f^* \) is free. If the first case is true, the
model reduces to the no coupon case, as zero face value implies no coupon distribution.

If the second case is true, i.e. \( \lambda^* = 1 \), we have the following: 
\[
p^* = \frac{r + 2f^*}{2} = \frac{r}{2} + f^*.
\]

From \( \frac{d\Pi_{\text{Total}}}{d\lambda} = 0 \) we have 
\[
2 \left( -rf^* + 2p^*f^* - f^{*2} \right) = \frac{b_1}{t} \text{ or } -rf^* + 2p^*f^* - f^{*2} = \frac{tb_1}{2}.
\]

Substituting \( p^* = \frac{r}{2} + f^* \) gives: 
\[
-rf^* + 2(\frac{r}{2} + f^*)f^* - f^{*2} = \frac{tb_1}{2}.
\]
Hence, we have 
\[
f^* = \sqrt{\frac{tb_1}{2}}.
\]
So, 
\[
p^* = \frac{r}{2} + f^* = \frac{r}{2} + \sqrt{\frac{tb_1}{2}}.
\]
Additionally, we have 
\[
x^* = \frac{f^*}{t} = \sqrt{\frac{b_1}{2t}} \text{ and } d^* = \frac{r - p^*}{t} = \frac{r - \sqrt{\frac{tb_1}{2}}}{2t} = \frac{r}{2} - \frac{\sqrt{tb_1}}{2t} = \frac{r}{2t} - \frac{b_1}{2t}.
\]
Hence the market coverage is 
\[
k^* = 2x + 2d = 2(x + d) = 2\left(\sqrt{\frac{b_1}{2t}} + \frac{r}{2t} - \sqrt{\frac{b_1}{2t}}\right) = \frac{r}{t}.
\]

Total profit will be: 
\[
\Pi^*_{\text{Total}} = \frac{2}{t} \left[ r(p^* - p^{*2} - \lambda^*rf^* + 2\lambda^*p^*f^* - \lambda^*f^{*2}) \right] - a_i - b_i \lambda^* \text{ or }
\]
\[
\Pi^*_{\text{Total}} = \frac{2}{t} \left[ \frac{r}{2} + \sqrt{\frac{tb_1}{2}} - \left( \frac{r}{2} + \sqrt{\frac{tb_1}{2}} \right)^2 - \frac{tb_1}{2} + \frac{r}{2} + \sqrt{\frac{tb_1}{2}} - \sqrt{\frac{tb_1}{2}} - \left( \sqrt{\frac{tb_1}{2}} \right)^2 \right] - a_i - b_i.
\]

After some algebra, we have \( \Pi^*_{\text{Total}} = \frac{r^2}{2t} - a_i - b_i \).

**A.2.2 Internet Changing Value Coupons with Free Perfect Information**

\[
\begin{align*}
\text{Max} & \quad \left[ \frac{2rp - p^2}{t} \right] \\
\text{Subject To} & \quad 0 \leq p \leq r \\
& \quad 0 \leq \frac{r - p}{t} \leq \frac{1}{2}
\end{align*}
\]
\[
p \leq \frac{1}{2} \left( \frac{r-p}{t} \right)
\]

F.O.C.

\[
\frac{d\Pi_{\text{total}}}{dp} = 0 = \frac{2r-2p^*}{t}
\]

Since \( \frac{d^2\Pi_{\text{total}}}{dp^2} = -\frac{2}{t} < 0 \) for all \( t > 0 \), the second order condition for maximization is also met. Hence we find the solution as \( p^* = r \) and \( \Pi_{\text{total}}^* = \frac{r^2}{t} \). We have

\[
d^* = \frac{r-p^*}{t} = \frac{r-r}{t} = 0.
\]

Let \( k \) be the percentage of the market that is covered, or simply the market coverage. Then, \( k^* = 2d^* + 2x_m = 0 + 2 \frac{p}{t} = \frac{2r}{t} \).

**A.2.3 Internet Fixed Face Value Coupons**

\[
\text{Max}_{p,f} \left[ \frac{2}{t} \left( rp - p^2 + pf - f^2 - b_z f \right) \right]
\]

Subject To

\[
0 \leq p \leq r
\]
\[
0 \leq f \leq p
\]
\[
0 \leq \frac{r-p}{t} \leq \frac{1}{2}
\]
\[
0 \leq \frac{f}{t} \leq \frac{1}{2} - \frac{r-p}{t}
\]

F.O.C.

\[
\frac{d\Pi_{\text{total}}}{dp} = 0 = \frac{2(r-2p^* + f^*)}{t}
\]

\[
\frac{d\Pi_{\text{total}}}{df} = 0 = \frac{2(p-2f^* - b_z)}{t}
\]
In order to check the second order conditions we need to calculate the Hessian matrix and we need to show that the Hessian matrix is at least negative semidefinite. The Hessian for this model is negative definite. Therefore the second order condition for maximization is satisfied. We also checked the Kuhn-Tucker conditions for the optimal values of the decision variables and they are satisfied. From \( \frac{d\Pi_{\text{Total}}}{dp} = 0 \) we have

\[
p^* = \frac{r + f^*}{2}.
\]

From \( \frac{d\Pi_{\text{Total}}}{df} = 0 \) we have \( p^* = 2f^* + b_2 \). Hence, \( f^* = \frac{r - 2b_2}{3} \),

\[
p^* = \frac{2r - b}{3} \quad \text{and} \quad x^* = \frac{f}{t} = \frac{r - 2b_2}{3t}.
\]

Additionally, \( d^* = \frac{r - p}{t} = \frac{r + b_2}{3t} \). Hence the market coverage is \( k^* = 2x + 2d = \frac{2}{3t}(2r - b_2) \). Total profit is

\[
\Pi_{\text{Total}}^* = \frac{2}{3t}\left[r^2 - rb_2 + b_2^2\right].
\]

A.2.4 Changing Face Value Internet Coupons to Targeted Consumers

\[
\max_{p,x_m}\left[\frac{2rp - 2p^2 + 2txpx_m - qx_m^2r^2 - 2tb_2x_m - ta_2q^2}{t}\right]
\]

Subject To

\[
0 \leq p \leq r
\]

\[
0 \leq f \leq p
\]

\[
0 \leq \frac{r - p}{t} \leq \frac{1}{2}
\]

\[
0 \leq x_m \leq \frac{1}{2} - \frac{r - p}{t}
\]

F.O.C.

\[
\frac{d\Pi_{\text{total}}}{dp} = 0 = \frac{2r - 4p^* + 2tx_m^*}{t}
\]

\[
\frac{d\Pi_{\text{total}}}{dx_m} = 0 = \frac{2tp^* - 2qx_m^*t^2 - 2tb_2}{t}
\]
In order to check to second order conditions we need to calculate the Hessian matrix and we need to show that the Hessian matrix is at least negative semidefinite. The Hessian for this model is negative semidefinite. Therefore the second order condition for maximization is satisfied. We also checked the Kuhn-Tucker conditions for the optimal values of the decision variables and they are satisfied. Hence the profit maximizing solution gives \( p^* = \frac{r-b_2}{2-q} \) and \( x_m^* = \frac{qr-2b_2}{qt(2-q)} \).

\[
\Pi_{\text{final}}^* = \frac{a_q t q^4 - 2a_q t q^5 - 2r b_2 q + r^2 q + 2b_2^2}{2tq(2-q)}. \]

We additionally have the following:

\[
d^* = \frac{r - qr + b_2}{t(2-q)}. \]

The market coverage is then equal to \( k^* = 2d^* + 2qx_m^* = \frac{2(r-b_2)}{t(2-q)}. \)
### APPENDIX B
#### NOTATION

Table 22 Notation for All Models

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>common reservation value</td>
</tr>
<tr>
<td>( p )</td>
<td>price</td>
</tr>
<tr>
<td>( t )</td>
<td>fit cost</td>
</tr>
<tr>
<td>( d )</td>
<td>distance of the marginal consumer from the available product</td>
</tr>
<tr>
<td>( d_i )</td>
<td>distance of consumer ( I ) from the available product</td>
</tr>
<tr>
<td>( k )</td>
<td>firm’s total market coverage (share)</td>
</tr>
<tr>
<td>( \Pi_{NC} )</td>
<td>profit from the no coupon segment</td>
</tr>
<tr>
<td>( \Pi_C )</td>
<td>profit from the coupon segment</td>
</tr>
<tr>
<td>( \Pi_{Total} )</td>
<td>total profit</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>coupon distribution intensity (fraction of consumers who get a paper coupon)</td>
</tr>
<tr>
<td>( C(.) )</td>
<td>cost of coupon distribution</td>
</tr>
<tr>
<td>( a )</td>
<td>coupon distribution cost parameter for paper coupon</td>
</tr>
<tr>
<td>( a_1 )</td>
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LIST OF REFERENCES


BIOGRAPHICAL SKETCH

Kutsal Dogan was born in Ankara, Turkey, in July of 1972. He received his Bachelor of Science degree in management engineering in 1993 from Istanbul Technical University in Istanbul, Turkey. He received the Master of Business Administration degree in 1997 from Virginia Tech in Blacksburg, Virginia. He is currently a fourth-year doctoral candidate in the Decision and Information Sciences Department at the University of Florida. He is specializing in information systems and expecting to finish his Ph.D. degree in August 2002.

Kutsal’s research and teaching interests are in the analytical modeling of electronic commerce and information systems. He is a member of Beta Gamma Sigma Business Honorary Society, Institute for Operations Research and Management Science (INFORMS), Association for Information Systems (AIS) and Decision Sciences Institute (DSI).